Name: ______________________________

PHYS3701 Intro QM I S24 Quiz #5 29 Feb 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Find the expectation value of the kinetic energy $p^2/2m$ in an eigenstate $|n\rangle$ of the simple harmonic oscillator in one dimension. Express your answer as a factor times the energy eigenvalue E_n. Use your result to infer (or otherwise calculate) the expectation value of the potential energy $m\omega^2x^2/2$. You are welcome to use any results from your homework that you would find useful, but refer to the relevant problem solution.

PHYS3701 Intro QM I Quiz #5 Solution

Find the expectation value of the kinetic energy $p^2/2m$ in an eigenstate $|n\rangle$ of the simple harmonic oscillator in one dimension. Express your answer as a factor times the energy eigenvalue E_n. Use your result to infer (or otherwise calculate) the expectation value of the potential energy $m\omega^2x^2/2$. You are welcome to use any results from your homework that you would find useful, but refer to the relevant problem solution.

We are asked to evaluate $\langle n|p^2/2m|n\rangle = \langle n|p^2|n\rangle/2m$. However, in HW#6 Problem 5, we calculated

$$\langle n|p^2|n\rangle = -\frac{m\hbar\omega}{2}\langle n|(aa - aa^\dagger - a^\dagger a + a^\dagger a^\dagger)|n\rangle = -\frac{m\hbar\omega}{2}[-(n+1) - n] = m\hbar\omega\left(n + \frac{1}{2}\right)$$

We also know, as we derived in class

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Therefore the expectation value of the kinetic energy is just

$$\frac{1}{2m}\langle p^2\rangle = \frac{\hbar\omega}{2}\left(n + \frac{1}{2}\right) = \frac{1}{2}E_n$$

This means that the expectation value of the potential energy must be the rest, namely also $E_n/2$. More formally

$$\langle H\rangle = E_n = \frac{1}{2m}\langle p^2\rangle + \frac{1}{2}m\omega^2\langle x^2\rangle \qquad \text{so} \qquad \frac{1}{2}m\omega^2\langle x^2\rangle = E_n - \frac{1}{2}E_n = \frac{1}{2}E_n$$

You could also derive this result, again using Problem 5 in HW#6, that is

$$\frac{1}{2}m\omega^2\langle n|x^2|n\rangle = \frac{1}{2}m\omega^2\frac{\hbar}{2m\omega}\langle n|(aa + aa^\dagger + a^\dagger a + a^\dagger a^\dagger)|n\rangle = \frac{\hbar\omega}{2}\left(n + \frac{1}{2}\right) = \frac{1}{2}E_n$$