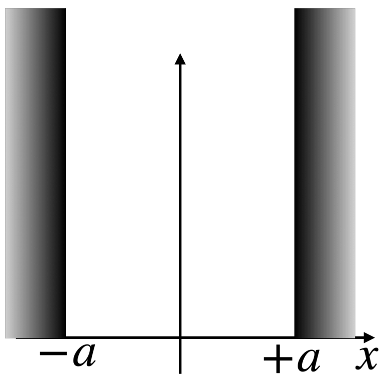
Name: ______________________________

PHYS3701 Intro QM I S24 Quiz #4 22 Feb 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

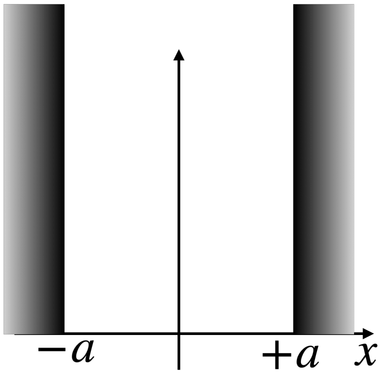


A particle of mass m is confined to an infinite square potential well in one dimension and that extends over $-a \leq x \leq a$. Writing the wave function as

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

show that the boundary conditions imply the energy levels are the same as we derived in class for an infinite square well extending over $0 \leq x \leq L$, and that $\psi(-x) = \pm\psi(x)$. *Hint: Recall how matrices solve simultaneous algebraic equations.*

PHYS3701 Intro QM I Quiz #4 Solution



A particle of mass m is confined to an infinite square potential well in one dimension and that extends over $-a \le x \le a$. Writing the wave function as

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

show that the boundary conditions imply the energy levels are the same as we derived in class for an infinite square well extending over $0 \le x \le L$, and that $\psi(-x) = \pm\psi(x)$. *Hint: Recall how matrices solve simultaneous algebraic equations.*

The wave function has to vanish at $x = \pm a$, so

$$\begin{aligned} Ae^{ika} + Be^{-ika} &= 0 \\ Ae^{-ika} + Be^{ika} &= 0 \end{aligned}$$

Written as a matrix equation for A and B, this means that we must have

$$\begin{vmatrix} e^{ika} & e^{-ika} \\ e^{-ika} & e^{ika} \end{vmatrix} = e^{2ika} - e^{-2ika} = 2i\sin(2ka) = 0$$

This implies that $2ka = n\pi$ for $n = 1, 2, 3\ldots$, so the energy levels are

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 (ka)^2}{2ma^2} = \frac{\hbar^2\pi^2}{8ma^2}n^2 = \frac{\hbar^2\pi^2}{2mL^2}n^2$$

where $L = 2a$, and this is the result we got in class.

Now $2ka = n\pi$ means $ka = (m+1/2)\pi$, $m = 1, 2, 3\ldots$ if n is odd, or $ka = m\pi$, $m = 1, 2, 3\ldots$ if n is even. For the case of n odd, we have

$$Ae^{im\pi}e^{i\pi/2} + Be^{-im\pi}e^{-i\pi/2} = i\cos(m\pi)[A - B] = 0$$

so $B = A$ in which case $\psi(x) = Ae^{ikx} + Ae^{-ikx}$ and $\psi(-x) = \psi(x)$. For the case of n even,

$$Ae^{im\pi} + Be^{-im\pi} = \cos(m\pi)[A + B] = 0$$

so $B = -A$ in which case $\psi(x) = Ae^{ikx} - Ae^{-ikx}$ and $\psi(-x) = -\psi(x)$.