Name: ____________________

PHYS3701 Intro QM I S24 Quiz #3 15 Feb 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Prove the following, where p is the canonically conjugate momentum to x:

$$[x, F(p)] = i\hbar \frac{dF}{dp}$$

You can do this by first proving that $[x, p^n] = i\hbar n p^{n-1}$ by showing it is true for $n = 1$ and then showing that if it is true for n, then it is true for $n + 1$, and then writing $F(p)$ as a Taylor series, that is $F(p) = \sum_n a_n p^n$ where the a_n are just some constants.

PHYS3701 Intro QM I Quiz #3 Solution

Prove the following, where p is the canonically conjugate momentum to x:

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Do this by following the same procedure as for Homework #4 Problem (3).

Proving that $[x, p^n] = i\hbar n p^{n-1}$ for $n = 1$ is just stating the canonical commutation relation, namely $[x, p] = i\hbar$. Writing it down for $n+1$ and using the relation for n gives

$$\begin{aligned}
[x, p^{n+1}] &= xp^{n+1} - p^{n+1}x \\
&= (xp)p^n - p^{n+1}x \\
&= (i\hbar + px)p^n - p^{n+1}x \\
&= i\hbar p^n + pxp^n - p^{n+1}x \\
&= i\hbar p^n + p(xp^n - p^n x) \\
&= i\hbar p^n + p[x, p^n] \\
&= i\hbar p^n + p(i\hbar n p^{n-1}) \\
&= i\hbar (n+1) p^n \qquad \text{QED}
\end{aligned}$$

Now it is straightforward to get the final result.

$$[x, F(p)] = \left[x, \sum_n a_n p^n\right] = \sum_n a_n \left[x, p^n\right] = i\hbar \sum_n n a_n p^{n-1} = i\hbar \frac{dF}{dp}$$