Name: _____

PHYS3701 Intro QM I S24 Quiz #2 8 Feb 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

A spin-1/2 system is governed by the Hamiltonian $H = \omega S_z$. If the system starts out in the state $|+\hat{\mathbf{y}}\rangle$, find the probability at some time t that a measurement finds the system in the state $|-\hat{\mathbf{z}}\rangle$. Explain why your answer does (or does not) make sense.

PHYS3701 Intro QM I Quiz #2 Solution

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The answer is a very straightforward application of the time evolution operator. The time evolved state is

$$\begin{aligned} |\alpha;t\rangle &= e^{-iHt/\hbar}|\alpha\rangle = e^{-i\omega S_z t/\hbar}\,|+\hat{\mathbf{y}}\rangle \\ &= e^{-i\omega S_z t/\hbar}\left[\frac{1}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle + \frac{i}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle\right] = \frac{e^{-i\omega t/2}}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle + i\frac{e^{i\omega t/2}}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle \end{aligned}$$

The probability to find the system in the $|-\hat{\mathbf{z}}\rangle$ state is

$$\left|i\frac{e^{i\omega t/2}}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

which is what you expect. Starting in the $|+\hat{\mathbf{y}}\rangle$ state, with a field in the z-direction, the spin precesses around the z-axis, with a changing linear combination of $|+\hat{\mathbf{y}}\rangle$ and $|-\hat{\mathbf{y}}\rangle$, which is the same as some other linear combination of $|+\hat{\mathbf{x}}\rangle$ and $|-\hat{\mathbf{x}}\rangle$. For any of these, the probability of measuring the spin in the z-direction is 1/2.