Name: ____________________

PHYS3701 Intro QM I S24 Quiz #1 1 Feb 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

A certain quantum mechanical system for a spin-1/2 particle is described by the state

$$|\alpha\rangle = a\,|+\hat{\mathbf{z}}\rangle + \frac{i}{\sqrt{3}}\,|-\hat{\mathbf{z}}\rangle$$

where a is a positive real number.

(a) Determine the value of a.

(b) Find the probability that the system would be measured to be in the state $|+\hat{\mathbf{x}}\rangle$.

(c) Calculate what you expect to find for the average $\langle S_z\rangle$ of a large number of measurements of the z-component of the spin vector.

PHYS3701 Intro QM I Quiz #1 Solution

A certain quantum mechanical system for a spin-1/2 particle is described by the state

$$|\alpha\rangle = a\,|{+}\hat{\mathbf{z}}\rangle + \frac{i}{\sqrt{3}}\,|{-}\hat{\mathbf{z}}\rangle$$

where a is a positive real number.

(a) Determine the value of a.

(b) Find the probability that the system would be measured to be in the state $|{+}\hat{\mathbf{x}}\rangle$.

(c) Calculate what you expect to find for the average $\langle S_z\rangle$ of a large number of measurements of the z-component of the spin vector.

The state has to be normalized, so

$$a^2 + \frac{1}{3} = 1 \qquad \text{therefore} \qquad a = \sqrt{\frac{2}{3}}$$

The probability is the square of the projection of the state onto the measured state, so

$$\begin{aligned}
|\langle{+}\hat{\mathbf{x}}|\,\alpha\rangle|^2 &= \left|\left[\frac{1}{\sqrt{2}}\langle{+}\hat{\mathbf{z}}| + \frac{1}{\sqrt{2}}\langle{-}\hat{\mathbf{z}}|\right]\left[\sqrt{\frac{2}{3}}\,|{+}\hat{\mathbf{z}}\rangle + \frac{i}{\sqrt{3}}\,|{-}\hat{\mathbf{z}}\rangle\right]\right|^2 \\
&= \left|\frac{1}{\sqrt{3}} + \frac{i}{\sqrt{6}}\right|^2 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}
\end{aligned}$$

The average measured value is just the sum of the different possible values multiplied by the probability for getting that possibility, so

$$\langle S_z\rangle = \left(\frac{\hbar}{2}\right)\left(\frac{2}{3}\right) + \left(-\frac{\hbar}{2}\right)\left(\frac{1}{3}\right) = \frac{\hbar}{6}$$