PHYS3701 Intro Quantum Mechanics I HW#13 Due 23 Apr 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) We showed in class that when $r \to 0$, the radial wave function for central potential problems can, in principle, have the form

$$R(r) = Ar^l + \frac{B}{r^{l+1}}$$

where A and B are constants. By integrating the probability flux $\vec{j} = (\hbar/m)\mathrm{Im}(\psi^* \vec{\nabla}\psi)$ over a small sphere around the origin, show that this would imply that the origin is a source of probability of both A and B are nonzero have a nonzero relative complex phase.

(2) Find the energy eigenvalues, and plot the radial eigenfunctions, for the following cases of an infinite spherical box of radius a:

(a) The lowest energy level (aka the ground state) with $l = 0$
(b) The first excited state with $l = 0$
(c) The second excited state with $l = 2$

(3) Find the lowest energy eigenvalues with $l = 0$ for a finite spherical box with radius a finite walls of height $V_0 = \hbar^2\beta^2/2ma^2$, where $\beta = 4$, 10, 25, and 100. (You need to do this numerically with MATHEMATICAor some other application.) Show that these results approach what you found in Problem (2) above.

(4) Construct all of the wave functions $\psi_{nlm}(r, \theta, \phi)$ for the eigenstates corresponding to the first excited state energy eigenvalue of the isotropic three dimensional harmonic oscillator for a particle of mass m and natural frequency ω. Using MATHEMATICA or some other graphing application, make a three dimensional plot of the probability density $\psi^*_{nlm}(r, \theta, \phi)\psi_{nlm}(r, \theta, \phi)$ for each of the wave functions.

(5) Look up the phenomenon called "magic numbers" in nuclear physics. Imagine that protons and neutrons move independently in an isotropic harmonic oscillator potential, and compare what you'd predict for the first five magic numbers to what is observed. Don't forget about the Paul Exclusion Principle, which I suppose you learned at some time.