PHYS3701 Intro Quantum Mechanics I HW#12 Due 16 Apr 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

I would encourage you to use MATHEMATICA or some other symbolic manipulation problem to work through at least some of these problems.

(1) Show by explicit calculation that the kinetic energy of two masses m_1 and m_2 is

$$\frac{1}{2m_1}\vec{p}_1^{\,2} + \frac{1}{2m_2}\vec{p}_2^{\,2} = \frac{1}{2M}\vec{P}^{\,2} + \frac{1}{2m}\vec{p}^{\,2}$$

where $M \equiv m_1 + m_2$, $m = m_1 m_2/M$, $\vec{P} = \vec{p}_1 + \vec{p}_2$, and $\vec{p} = (m_2\vec{p}_1 - m_1\vec{p}_2)/M$.

(2) A three-dimensional spatial wave function over all space has the form

$$\psi(\vec{r}) = N(x + y + 2z)e^{-\alpha^2 r^2}$$

where α is a real constant.

(a) Find the normalization constant N. (You can assume it is real.)
(b) Determine the possible results from a measurement of L_z, and the probabilities that they are in fact the result of a measurement.
(c) Determine the possible results from a measurement of $\vec{L}^{\,2}$, and the probabilities that they are in fact the result of a measurement.

(3) Defining $L_\pm \equiv L_x \pm iL_y$ and the expression we derived for $\langle\vec{r}'|\vec{L}|\alpha\rangle$, show that

$$\langle\vec{r}'|L_\pm|\alpha\rangle = -i\hbar e^{\pm i\phi}\left(\pm i\frac{\partial}{\partial\theta} - \cot\theta\frac{\partial}{\partial\phi}\right)\langle\vec{r}'|\alpha\rangle$$

(4) Use the result of Problem **(3)** to show that

$$\langle\vec{r}'|\vec{L}^{\,2}|\alpha\rangle = -\hbar^2\left[\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]\langle\vec{r}'|\alpha\rangle$$

(5) A certain spherical harmonic is given by

$$Y_\ell^m(\theta,\phi) = \frac{3}{8}\sqrt{\frac{5}{2\pi}}e^{-2i\phi}\sin^2\theta\left(7\cos^2\theta - 1\right)$$

(a) Show that this function is properly normalized.
(b) Determine the values of ℓ and m. You might want to use the result of Problem **(4)**, along with the analogous result for the operator L_z.