PHYS3701 Intro Quantum Mechanics I HW#10 Due 2 Apr 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

There is only one problem for this week's assignment.

Write a program in MATHEMATICA or some other language to simulate Grover's algorithm. Use as large a number n of qubits as you think you can manage on your classical, digital computer. (It might be more than you would expect.) Remember that this means you will be dealing with column vectors of length $N = 2^n$, and with $N \times N$ matrices.

You should build the $N \times N$ Hadamard matrix using the tensor product of 2×2 matrices, and show that it creates the equal-superposition state from the $|\mathbf{0}\rangle$ state. You can build the matrix called "D" by hand instead of constructing it from gates. Remember that the elements are $D_{ij} = -\delta_{ij} + 2/N$. For the oracle, all you need to do is pick a target state, and change the sign of that element.

A simple way to check that you've made the equal-superposition state, and to watch the result of each Grover iteration, is to just plot the coefficients of each of the qubits, which is contained in your one N-dimensional qubit. In MATHEMATICA you can do this with ListPlot.

It is tedious to have to put in each Grover iteration by hand. You can try that to start, and watch what happens to the coefficient of your target bit for the first few iterations. But to do a large number of operations, you want to put this in a loop. In MATHEMATICA, I think the simplest way to do this is with For.

What does your array look like for $\sqrt{N}$ iterations? Confirm that this is more or less what you expect. You might find it interesting to also look at what is happening to the other coefficients.

It's only a suggestion, but if you want to make an animation of how the amplitudes change on the qubits with each iteration, that might be a cool demonstration.