PHYS3701 Intro Quantum Mechanics I HW#9 Due 26 Mar 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

Note: Most or all of these problems are worked out handily in MATHEMATICA.

(1) Show that the Pauli matrix σ_x has the effect of a NOT gate by showing that it gives the expected result on the $|\pm\hat{\mathbf{z}}\rangle$ representations of the states $|+\hat{\mathbf{z}}\rangle = |0\rangle$ and $|-\hat{\mathbf{z}}\rangle = |1\rangle$. Then form the tensor product $\sigma_x \otimes \sigma_x$ and show that it has the expected result on the $|\pm\hat{\mathbf{z}}\rangle$ representations of each of the four states $|\pm\hat{\mathbf{z}}\rangle \otimes |\pm\hat{\mathbf{z}}\rangle$.

(2) A "controlled NOT" gate for two qubits can be constructed as a 4×4 matrix of 2×2 matrices with $\underline{\underline{1}}$ and $\underline{\underline{\sigma}}_x$ along the diagonal and zeros otherwise. Show that a CNOT gate flips the second qubit if the first qubit is $|0\rangle$, but does nothing if the first qubit is $|1\rangle$.

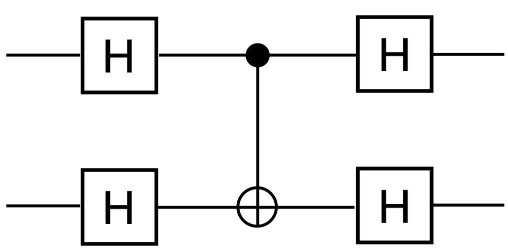
(3) The single qubit Hadamard gate is represented in the $|\pm\hat{\mathbf{z}}\rangle$ basis as

$$\underline{\underline{H}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

If we interpret $|0\rangle$ and $|1\rangle$ as $|\pm\hat{\mathbf{z}}\rangle$ respectively, show how we can use rotations to realize a Hadamard gate. Can you find a solution that does not introduce an overall phase factor?

(4) Show that the two-qubit Hadamard gate $H \otimes H$ acting on the two-qubit state $|0\rangle \otimes |0\rangle$ results in a "fully entangled" state of two qubits. That is, a state which cannot be written simply as a linear combination of one of the qubits times either $|0\rangle$ or $|1\rangle$.

(5) Find the 4×4 matrix representation (in the $|\pm\hat{\mathbf{z}}\rangle$ basis) for the following two-qubit gate constructed from four Hadamard gates and a CNOT gate:



Prove that your construction is a unitary transformation.