PHYS3701 Intro Quantum Mechanics I HW#8 Due 19 Mar 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Recall Problem (4) from Homework #1. Show that the state vector

$$|+\hat{\mathbf{n}}\rangle = \cos\frac{\theta}{2}\,|+\hat{\mathbf{z}}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-\hat{\mathbf{z}}\rangle$$

can be obtained by rotating the state $|+\hat{\mathbf{z}}\rangle$ by an angle θ about the y-axis, and then by an angle ϕ about the z-axis. You can approach this using the rotation operators, or by using the matrix representation in the $|\pm\hat{\mathbf{z}}\rangle$ basis; I'm not sure which one is easiest.

(2) The spin-dependent part of the Hamiltonian for a hydrogen atom (proton plus electron) in an external magnetic field $\vec{B} = B\hat{\mathbf{z}}$ is

$$H = \frac{2A}{\hbar^2}\vec{S}_e \cdot \vec{S}_p + \omega S_{e_z}$$

where A is a positive constant and $\omega = geB/2mc$. Find the energy eigenvalues, and their expressions to lowest non-vanishing order for the cases (a) $A \gg \hbar\omega$ and (b) $A \ll \hbar\omega$. The calculations for the eigenvalues and their limits is not hard to do by hand, but you are welcome to resort to MATHEMATICA or some other app if you like.

(3) Consider a spin-3/2 system with the four states $|3/2, \pm 3/2\rangle$ and $|3/2, \pm 1/2\rangle$ made up from three spin-1/2 particles. Using the operator $S_z = S_{1_z} + S_{2_z} + S_{3_z}$, explain why we must have

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = |+\hat{\mathbf{z}}, +\hat{\mathbf{z}}, +\hat{\mathbf{z}}\rangle$$

Then use the operator S_- to similarly construct the other three states.

(4) Express the two electron spin-one state $|\alpha\rangle = |+\hat{\mathbf{z}}, +\hat{\mathbf{z}}\rangle$ in terms of the four states $|+\hat{\mathbf{x}}, +\hat{\mathbf{x}}\rangle$, $|+\hat{\mathbf{x}}, -\hat{\mathbf{x}}\rangle$, $|-\hat{\mathbf{x}}, +\hat{\mathbf{x}}\rangle$, and $|-\hat{\mathbf{x}}, -\hat{\mathbf{x}}\rangle$. Calculate the probability that a measurement of the x-direction spins of an electron pair in the state $|\alpha\rangle$ yields a result where the two electrons have spins in the opposite direction.

(5) Two spin-1/2 particles are emitted from the spin-one state $|+\hat{\mathbf{z}}, +\hat{\mathbf{z}}\rangle$ and move in opposite directions when they are measured independently by observers A and B who make measurements of the spins in the x-direction. Find the probabilities that A and B determine the two particles to be in the states $|1, +1\rangle_x$, $|1, 0\rangle_x$, and $|1, -1\rangle_x$.