# PHYS3701 Intro Quantum Mechanics I HW#6 Due 26 Feb 2024

*This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.*

**(1)** A particle of mass $m$ is bound in a one-dimensional finite square well of height $V_0$. Assume that the well extends over the range $-a \leq x \leq a$. Remember that the ground (first excited) state will be even (odd) parity.

(a) Show that there will always be a solution for a ground state eigenvalue, but there may not be a solution for any other states if $V_0$ is too small.

(b) Find the energy eigenvalues and plot their wave functions for the ground and first excited states assuming that $V_0 = 1.2(\hbar^2\pi^2/8ma^2)$. You will need to numerically solve two transcendental equations, one for the each of the two states. (*Hint*: In addition to matching boundary conditions, you can easily derive a formula for $(ka)^2 + (qa)^2$ where $k$ and $q$ are the wave numbers inside and outside the well.) Express the energy eigenvalues as a numerical factor times $V_0$.

**(2)** Use induction to prove that the normalized states of the quantum harmonic oscillator are given by

$$|n\rangle = \frac{1}{\sqrt{n!}} \left(a^\dagger\right)^n |0\rangle$$

**(3)** Find $\langle x \rangle$ and $\langle p \rangle$ as a function of time for the initial state

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\delta}}{\sqrt{2}}|1\rangle$$

where $\delta$ is a real number. Explain why this makes sense classically by taking the time derivative of $\langle x \rangle$ and showing that it has the expected relationship to $\langle p \rangle$, and interpret the physical meaning of the phase $\delta$.

**(4)** The goal here is to find the harmonic oscillator normalized eigenfunction $\langle x|3\rangle$ using properties of the creation and annihilation operators.

(a) Find the harmonic oscillator ground state wave function $\langle x|0\rangle$ by considering $\langle x|a|0\rangle$ and solving the resulting simple differential equation.

(b) Now use the result from Problem (2) above to find $\langle x|3\rangle$ by building up from $\langle x|0\rangle$. Show that your result agrees with the result from solving the Schrödinger equation. Integrate to prove the normalization is correct. (The calculus in this part is messy. I suggest that you use Mathematica.)

**(5)** Find $\Delta x$ and $\Delta p$ for the harmonic oscillator eigenstate $|n\rangle$ and compare the result to Heisenberg's uncertainty principle. Show that $n = 0$ yields the minimum possible result for the uncertainty product.