PHYS3701 Intro Quantum Mechanics I HW#1 Due 23 Jan 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) A beam of silver atoms is created by heating a vapor to 1500°C, and selecting atoms with a velocity close to the thermal mean value. The beam moves through a 1.5 m long magnetic field with a vertical gradient 8 T/m, and impinges a screen 2 m downstream of the end of the magnet. Assuming the silver atom has spin-1/2 with a magnetic moment of one Bohr magneton, find the separation distance in mm of the two states on the screen. *This is the calculation Stern and Gerlach had to do in order to design their experiment.*

(2) In class we wrote down the states $|\pm\hat{\mathbf{x}}\rangle$ and $|\pm\hat{\mathbf{y}}\rangle$ in terms of the two states $|\pm\hat{\mathbf{z}}\rangle$. Using these expressions, show that

(a) $|+\hat{\mathbf{x}}\rangle$ is orthogonal to $|-\hat{\mathbf{x}}\rangle$

(b) $|+\hat{\mathbf{y}}\rangle$ is orthogonal to $|-\hat{\mathbf{y}}\rangle$

(c) The probability of measuring an electron to have its spin pointing in the $+\hat{\mathbf{y}}$ direction, when the electron in fact is in the $|-\hat{\mathbf{z}}\rangle$ state, is 1/2

(3) A spin-1/2 particle, say an electron, exists in the state

$$|\alpha\rangle = \frac{i}{2}|+\hat{\mathbf{z}}\rangle - \frac{\sqrt{3}}{2}|-\hat{\mathbf{z}}\rangle$$

What is the probability that a measurement of spin the $-\hat{\mathbf{y}}$ direction gives the value $\hbar/2$?

(4) For an arbitrary unit vector $\hat{\mathbf{n}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$, I claim that

$$|+\hat{\mathbf{n}}\rangle = \cos\frac{\theta}{2}|+\hat{\mathbf{z}}\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\hat{\mathbf{z}}\rangle$$

is the state for which a measurement of spin in the $+\hat{\mathbf{n}}$ direction will always give $\hbar/2$. Construct the corresponding state $|-\hat{\mathbf{n}}\rangle$ in terms of $|+\hat{\mathbf{z}}\rangle$ and $|-\hat{\mathbf{z}}\rangle$ by forcing it to be normalized, orthogonal to $|+\hat{\mathbf{n}}\rangle$, and with a positive, real coefficient of $|+\hat{\mathbf{z}}\rangle$. If you recognize that θ and ϕ are just the normal polar angles in three dimensions, can you see why you could have easily guessed your result for $|-\hat{\mathbf{n}}\rangle$?

(5) Suppose you made a very large number of measurements of the spin in the $\hat{\mathbf{z}}$ direction for a bunch of electrons all in the state

$$|\alpha\rangle = \sqrt{\frac{1}{3}}|+\hat{\mathbf{z}}\rangle + \sqrt{\frac{2}{3}}|-\hat{\mathbf{z}}\rangle$$

In terms of $\hbar$, what would be the average value of all of your measurements?