Name: _____

PHYS3101 Analytical Mechanics S23 Quiz #13 7 Dec 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

A projectile of mass m flies through the air without drag close to the Earth's surface. Using a coordinate system where x is horizontal and y is upward, write down the Hamiltonian function $\mathcal{H}(x, y, p_x, p_y)$ and show that Hamilton's equations reduce to the correct second order differential equations for $x(t)$ and $y(t)$.

PHY3101 Analytical Mechanics Quiz #13 Solution

A projectile of mass m flies through the air without drag close to the Earth's surface. Using a coordinate system where x is horizontal and y is upward, write down the Hamiltonian function $\mathcal{H}(x, y, p_x, p_y)$ and show that Hamilton's equations reduce to the correct second order differential equations for $x(t)$ and $y(t)$.

There are no explicit time dependences, so the Hamiltonian is just the total energy.

$$\mathcal{H}(x, y, p_x, p_y) = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + mgy$$

Hamilton's equations for x are

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial p_x} = \frac{p_x}{m} \qquad \text{and} \qquad \dot{p}_x = m\ddot{x} = -\frac{\partial \mathcal{H}}{\partial x} = 0 \qquad \text{so} \qquad \ddot{x} = 0$$

which we know to be correct since there are no forces in the x-direction.
Hamilton's equations for y are

$$\dot{y} = \frac{\partial \mathcal{H}}{\partial p_y} = \frac{p_y}{m} \qquad \text{and} \qquad \dot{p}_y = m\ddot{y} = -\frac{\partial \mathcal{H}}{\partial y} = -mg \qquad \text{so} \qquad \ddot{y} = -g$$

which is also exactly what you expect, that is, the vertical acceleration is just $-g$.