Name: _____

PHYS3101 Analytical Mechanics S23 Quiz #11 9 Nov 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Two particles A and B with four-momenta p_A and p_B collide. Show that the total energy E_{CM} in the center-of-mass frame is given by

$$E_{\mathrm{CM}}^2 = (p_A + p_B)^2 c^2$$

PHY3101 Analytical Mechanics Quiz #11 Solution

Two particles A and B with four-momenta p_A and p_B collide. Show that the total energy $E_{\rm CM}$ in the center-of-mass frame is given by

$$E_{\rm CM}^2 = (p_A + p_B)^2 c^2$$

Write the four-momenta in the CM frame as

$$p_A = \left(\frac{E_A}{c}, \vec{p}_A\right) \qquad \text{and} \qquad p_B = \left(\frac{E_B}{c}, \vec{p}_B\right)$$

where the definition of the CM frame means that $\vec{p}_A + \vec{p}_B = 0$. Then

$$(p_A + p_B)^2 = \left(\frac{E_A + E_B}{c}, \vec{p}_A + \vec{p}_B\right)^2 = \left(\frac{E_A + E_B}{c}, \vec{0}\right)^2 = \frac{(E_A + E_B)^2}{c^2}$$

However, $E_A + E_B = E_{\rm CM}$, so

$$E_{\rm CM}^2 = (p_A + p_B)^2 c^2$$