Name: _____

PHYS3101 Analytical Mechanics S23 Quiz #8 19 Oct 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

A flat square plate of mass M and side length L rotates with angular velocity ω about an axis perpendicular to the plane of the plate and passing through one corner. Find the kinetic energy of the plate in terms of M, L, and ω. You are welcome to use your homework or a Google search to find the moment of inertia of the plate, but make it clear from where you get your answer.

PHY3101 Analytical Mechanics Quiz #8 Solution

A flat square plate of mass M and side length L rotates with angular velocity ω about an axis perpendicular to the plane of the plate and passing through one corner. Find the kinetic energy of the plate in terms of M, L, and ω. You are welcome to use your homework or a Google search to find the moment of inertia of the plate, but make it clear from where you get your answer.

The kinetic energy is $I\omega^2/2$ where $I = I_{zz}$ is the component of the inertia tensor in a coordinate system where the plate is in the xy plane, and the z-axis passes through the origin. For your homework, you found for a rectangular plate of sides a and b,

$$I_{zz} = \frac{M}{3}(a^2 + b^2) = \frac{2}{3}ML^2$$

I confirmed this answer by Googling "moment of inertia plate corner" and I found

https://www.toppr.com/ask/en-us/question/the-moment-of-inertia-of-a-thin-uniform-rectangular-plate/#

It is also not difficult to find the moment of inertia through the symmetry axis (which passes through the center of mass), namely $ML^2/6$. From the parallel axis theorem, then, the moment of inertia about a corner is this plus $M\Delta^2$ where $\Delta = L\sqrt{2}/2$, that is

$$I_z z = \frac{1}{6}ML^2 + M\frac{2L^2}{4} = \frac{1}{12}ML^2(2+6) = \frac{2}{3}ML^2$$

In any case, the kinetic energy is

$$T = \frac{1}{2}\frac{2}{3}ML^2\,\omega^2 = \frac{1}{3}ML^2\omega^2$$