Name: ____________________

PHYS3101 Analytical Mechanics S23 Quiz #7 12 Oct 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Calculate the inertia tensor for a uniform sphere of mass M and radius R for a coordinate system with origin located at the center of the sphere, and then find the principle moments of inertia. You may define the x, y, and z axes any way you want, so long as they are orthogonal and right handed.

PHY3101 Analytical Mechanics Quiz #7 <u>Solution</u>

Calculate the inertia tensor for a uniform sphere of mass M and radius R for a coordinate system with origin located at the center of the sphere, and then find the principle moments of inertia. You may define the x, y, and z axes any way you want, so long as they are orthogonal and right handed.

The sphere is 100% symmetric, so the inertia tensor is already diagonal and all three diagonal elements are the same. The result is well known, but also very easy to derive, using spherical coordinates, namely (using the substitution $\mu = \cos\theta$)

$$\begin{aligned}
\lambda &= \int_{\text{Sphere}} \rho^2\, dm = \frac{M}{4\pi R^3/3} \int_{\text{Sphere}} \rho^2\, dV \\
&= \frac{3M}{4\pi R^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^R (r\sin\theta)^2\, r^2 dr \\
&= \frac{3M}{2R^3} \int_0^{\pi} \sin^2\theta\, \sin\theta\, d\theta \int_0^R r^2\, r^2 dr \\
&= \frac{3M}{2R^3} \int_{-1}^{1} (1-\mu^2)\, d\mu\, \frac{R^5}{5} \\
&= \frac{3}{10} M R^2 \left[\mu - \frac{1}{3}\mu^3\right]_{-1}^{1} \\
&= \frac{3}{10} M R^2 \frac{4}{3} \\
&= \frac{2}{5} M R^2
\end{aligned}$$