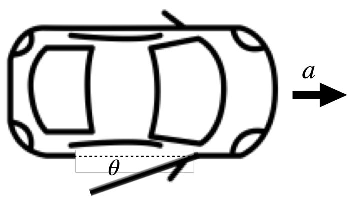
Name: ____________________

PHYS3101 Analytical Mechanics S23 Quiz #6 5 Oct 2023

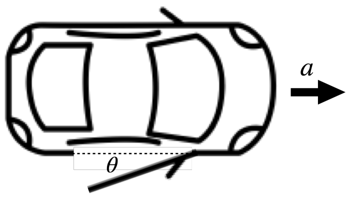
You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.



A car on a flat, horizontal street accelerates from rest with an acceleration $a = 2\ \mathrm{m/s^2}$. One of the doors is initially open an angle $\theta \ll 1$. The door is 1 m long and has a mass $m = 20$ kg. Assuming the door is of uniform density, find the amount of time it takes for the door to close.

PHY3101 Analytical Mechanics Quiz #6 Solution



A car on a flat, horizontal street accelerates from rest with an acceleration $a = 2$ m/s^2. One of the doors is initially open an angle $\theta \ll 1$. The door is 1 m long and has a mass $m = 20$ kg. Assuming the door is of uniform density, find the amount of time it takes for the door to close.

This is a pendulum problem, except that instead of gravity ("mg") pulling downward on the mass, there is a pseudo force ma acting towards the left, in the car's frame of reference. The door's mass is irrelevant, but since the density is uniform we can take the length of the pendulum to be one half of the door's length. The door closes in one-fourth of a period, so

$$\text{time} = \frac{1}{4} 2\pi \sqrt{\frac{\ell}{a}} = \frac{1}{4} 2\pi \sqrt{\frac{1/2 \text{ m}}{2 \text{ m/s}^2}} = \frac{\pi}{4} \text{ sec}$$

The solution above is incorrect! The period of a physical pendulum is not equal to that of a point mass pendulum with all the mass at the CM. This does not take into account the forces that keep the rod rigid. Following is the correct derivation for a physical pendulum.

Analyze a stiff rod of length L and mass M in terms of the torque from gravity on the CM about the pivot point. That is, the torque $\tau = I\alpha$ where $I = ML^2/3$, $\alpha = \ddot{\theta}$, and $\tau = -(MgL/2)\sin\theta$. Then

$$-\frac{1}{2} MgL \sin\theta = \frac{1}{3} ML^2 \ddot{\theta} \qquad \text{so} \qquad \ddot{\theta} = -\frac{2}{3}\frac{g}{L}\sin\theta \approx -\frac{2}{3}\frac{g}{L}\theta \equiv -\omega^2\theta$$

for $\theta \ll 1$. Therefore, the time it takes the car door to close is

$$\text{time} = \frac{1}{4} 2\pi \sqrt{\frac{3}{2}\frac{\ell}{a}} = \frac{\pi}{4}\sqrt{\frac{3}{2}} \text{ sec}$$