Name: _____

PHYS3101 Analytical Mechanics S23 Quiz #1 31 Aug 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

An object of mass m is fired vertically upward from the Earth's surface with speed v_0. It experiences a linear drag force with magnitude bv where b is a constant and v is the velocity of the object. Assuming that the Earth's gravity exerts a constant force throughout the flight, find the time T at which the object reaches its maximum height, in terms of m, b, v_0, and the acceleration g from gravity.

PHY3101 Analytical Mechanics Quiz #1 Solution

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See Sec.2.2 in Taylor or Sec.3.2.4 in Concepts. The equation of motion is

$$m\dot{v} = m\frac{dv}{dt} = -mg - bv \qquad \text{with} \qquad v = v_0 \text{ at } t = 0 \qquad \text{and} \qquad v = 0 \text{ at } t = T$$

The rest is straightforward algebra and integration:

$$\begin{aligned} dv &= -g\left(1 + \frac{b}{mg}v\right) dt \\ \int_{v_0}^{0} \frac{dv}{1 + (b/mg)v} &= -g\int_0^T dt \\ \frac{mg}{b}\log\left(1 + \frac{b}{mg}v\right)_{v_0}^{0} &= -gT \\ T &= \frac{m}{b}\log\left(1 + \frac{b}{mg}v_0\right) \end{aligned}$$