PHYS3101 Analytical Mechanics Homework #13 Due 5 Dec 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

This assignment is on nonlinear dynamics and chaos as demonstrated by the Damped Driven Pendulum (DDP). You should execute, and play around, with the notebook provided on the course web page that goes with Chapter 12 in Taylor. You are welcome to borrow code from that notebook for this assignment.

(1) Reproduce Figure 12.7 in Taylor, namely two solutions for the DDP with the same drive strength and damping parameter, and the initial condition $\dot{\phi}(0) = 0$, but one solution for $\phi(0) = 0$ and the other for $\phi(0) = -\pi/2$. This example demonstrates that for a nonlinear system, the behavior can be wildly different for different initial conditions. Animate the two solutions, and compare them. (It would be more fun to do this with another person, and start the two animations at the same time to watch and compare in real time.)

(2) Using the code that reproduces Figure 12.4 in Taylor, a DDP with our standard frequencies and damping parameter and with drive strength $\gamma = 1.06$, find solutions for the two initial conditions $\phi(0) = \pi/2$ and $\phi(0) = -\pi/2$, both with $\dot{\phi}(0) = 0$. Plot all three solutions for $0 \le t \le 10$, or longer. Do all three approach the same solution after some period of time? You may need to remember that $\phi(t)$ is the same as $\phi(t) + 2\pi n$ for some integer n.

(3) The notebook for class demonstrates chaotic behavior when the drive strength $\gamma = 1.105$ with our other standard parameters. Increasing the drive strength to $\gamma = 1.503$ (Taylor Figures 12.15(a), 12.15(b), and 12.16) continues chaotic motion, but the motion is qualitatively very different. Reproduce these three figures. (Note that Figure 12.15 uses $\phi(0) = -\pi/2$.) You might see deviation from Taylor's figure after long times because of numerical precision, but you can consider using the option PrecisionGoal in NDSolve. Make an animation of these conditions, and watch the pendulum flip directions near $t \approx 17$.

(4) It happens that periodicity can be restored with driving strengths well past the onset of chaos. Set up and solve the DPP using a driving strength $\gamma = 1.3$ and our standard frequencies and damping parameter, with initial conditions $\phi(0) = \dot{\phi}(0) = 0$. Plot the solution and comment. (The animation might be fun to watch.) Show that the solution is in fact periodic at long times. You may need to subtract a linear function that looks something like $2\pi(t - t_0)$ to make the periodicity clear.

(5) This problem is an example of a *logistics map*, a mathematical example of nonlinearity which shows many of the same features as the DDP. See Taylor 12.9. Consider a set of numbers $\{x_0, x_1, x_2, \ldots, x_\infty\}$ is defined by the "sine map" $x_{i+1} = f(x_i)$ where $f(x) = r\sin(\pi x)$, an obviously nonlinear function. Find and plot the values of x_i for i up to some number, say $i_{\max} = 20$ to start, for $x_0 = 0.8$ and $r = 0.60$, 0.79, 0.85, and 0.865, and show that these values of r form a period-doubling cascade, similar to what happens in Figure 12.8 in Taylor. (You will likely find it useful to use the RecurrenceTable function in MATHEMATICA.)