PHYS3101 Analytical Mechanics Homework #11 Due 14 Nov 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) The MOLLER experiment at Jefferson Lab will measure elastic electron-electron, i.e. Møller, scattering with an electron beam impinging on a 125 cm long liquid hydrogen target. What is the target density of electrons in units of cm^{-2}? If the Møller scattering cross section is 60 μbarn, find the scattering rate if the beam current is 65 μA? (You will need to look up the density of liquid hydrogen.)

(2) The differential cross section for scattering 6.5-MeV α particles at 120° off a silver nucleus is about 0.5 barns/sr. If a total of 10^{10} α particles impinge on a silver foil of thickness 1 μm and if we detect the scattered particles using a counter of area 0.1 mm^2 at 120° and 1 cm from the target, about how many scattered α particles should we expect to count? Silver has a density of 10.5 g/cm^3, and atomic mass of 108.

(3) Our definition of the scattering cross section, $N_{\text{sc}} = N_{\text{inc}} n_{\text{tar}} \sigma$ applies to an experiment using a narrow beam of projectiles all of which pass through a wide target assembly. Experimenters sometimes use a wide incident beam, which completely engulfs a small target assembly. Show that in this case $N_{\text{sc}} = n_{\text{inc}} N_{\text{tar}} \sigma$ where n_{inc} is the density (number/area) of the incident beam, viewed head-on, and N_{tar} is the total number of targets in the target assembly.

(4) A particle of mass m_1 and total energy $E = T + m_1c^2$ scatters by an angle θ_{Lab} from a stationary particle of mass m_2. Use relativistic kinematics to derive an expression between the scattering angle θ_{CM} in the center-of-momentum, and θ_{lab}. In the case where the incident particle is non-relativistic ($T \ll m_1c^2$) show that

$$\tan\theta_{\text{Lab}} = \frac{\sin\theta_{CM}}{\lambda + \cos\theta_{\text{CM}}} \qquad \text{where} \qquad \lambda \equiv \frac{m_1}{m_2}$$

This problem is more difficult than I thought. I can see my way through to a solution, but it is an arduous path and I don't think it is particularly enlightening. It's not so difficult, though, if you start with the assumption that the motion is non-relativistic, but this is essentially done in Taylor Section 14.8, which I didn't cover in class. So, let's skip this problem and this week you get a break, only four homework problems instead of five.

(5) Consider the non-relativistic scattering of two particles of equal mass. First, using the result of the previous problem, show that $\theta_{\text{Lab}} = \theta_{\text{CM}}/2$. Then prove that

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Lab}} = 4\cos\theta_{\text{Lab}} \left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}$$

Now, given that in the CM frame, the differential cross section is $R^2/4$ where $R = R_1 + R_2$ where R_1 and R_2 are the radii of the two masses, integrate over all directions to verify that the total cross section in the lab frame is πR^2, as it has to be.