# PHYS3101 Analytical Mechanics Homework #9 Due 31 Oct 2023

*This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.*

**(1)** We solved in class the problem of the double pendulum. (See also Taylor Section 11.4.) Specialize to the case of equal masses and equal lengths. Use this solution to write the (two-dimensional) vector $\underline{\phi}(t)$ as a sum over the normal mode vectors with coefficients $\xi_i(t)$. Turn this around to determine the $\xi_i(t)$ in terms of $\phi_1(t)$ and $\phi_2(t)$, and then show that the $\xi_i(t)$ oscillate with the eigenfrequencies $\omega_i$ by deriving the differential equations for the $\xi_i(t)$ from those for $\phi_1(t)$ and $\phi_2(t)$.

**(2)** A mass $m$ moves in one horizontal direction $x$ on a frictionless track. The mass is connected to a spring with stiffness $k$, which is itself attached to a fixed wall. (So far, this is a very familiar, simple problem.) Now assume that the spring is not massless, but has a total mass $\mu$, uniformly distributed along its length, even as it stretches. Find the Hamiltonian $\mathcal{H}(x,p)$ and solve Hamilton's equations to find the oscillation frequency $\omega$ in terms of $m$, $\mu$, and $k$. (Remember that the spring is continuous and a small piece of it that is close to the wall is moving more slowly than a small piece that is closer to the mass.)

**(3)** A bead of mass $m$ moves without friction along a curved wire that lies entirely in the vertical plane. The shape of the wire is given by the function $y = h(x)$. Find the Hamiltonian $\mathcal{H}(x,p)$ and show that Hamilton's equations give the result is the same as Newton's Second Law in terms of a position variable $s$ that measures distance along the wire. (My apologies for the somewhat messy algebra and calculus.)

**(4)** In class, we used the Lagrangian approach to solve the problem of a bead of mass $m$ constrained to a circular wire hoop of radius $R$ which itself rotated about a vertical axis with angular velocity $\omega$. (See also Taylor Example. 7.6, with Figure 7.9.) Construct the Hamiltonian $\mathcal{H}(\theta, p_\theta)$ and comment on what is peculiar about it. Then show that Hamilton's equations lead to the same differential equation for $\ddot{\theta}$, namely Taylor (7.69).

**(5)** In class, we showed that the "potential energy" term in a Lagrangian for a particle with charge $q$ moving in a region of magnetic field $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$ is given by $q\dot{\mathbf{r}} \cdot \mathbf{A}$. (See also Taylor Section 7.9. I will stick with SI units here for the electromagnetic quantities.) Use this to find the Hamiltonian $\mathcal{H}(\mathbf{r}, \mathbf{p})$ for a particle with charge $q$ and mass $m$ in a magnetic field $\mathbf{B}$ and an electric field $\mathbf{E} = -\boldsymbol{\nabla} V$, and show that Hamilton's equations reduce to the Lorentz force law, namely $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$, where $\mathbf{v}$ is the velocity vector of the mass.