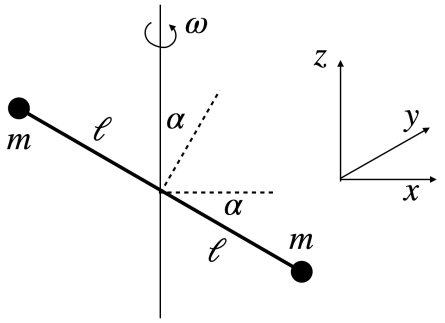
PHYS3101 Analytical Mechanics Homework #6 Due 10 Oct 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) The figure shows two equal masses m at the ends of a massless rod of length 2ℓ, rotating with angular velocity ω about an axis which passes through the center of mass. The normal vector to the rod makes an angle α with respect to the axis of rotation. At the instant shown, the rod lies in the xz plane. Use the coordinate system shown for the following calculations:



(a) Find all nine components of the inertia tensor for this coordinate system. **(b)** Find the (vector) angular momentum for the configuration as shown. **(c)** Find the kinetic energy for the configuration as shown. **(d)** Calculate the principal moments of inertia, and **(e)** find the principal axes for this configuration.

(2) Find the moment of inertia about the z-axis for a uniform ellipsoid whose surface is given by $(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$.(You can use MATHEMATICA if you want, but it's probably easier to just do the necessary integrals with a simple change of variables and exploiting symmetry.) Check your answer against the result for a sphere with $a = b = c = R$.

(3) Consider a top consisting of a uniform cone spinning freely about its tip at 1800 rpm. If its height is 10 cm and its base radius 2.5 cm, at what angular velocity will it precess?

(4) A rigid body is rotating freely, subject to zero torque. Use Euler's equations to prove that the magnitude of the angular momentum $\mathbf{L}$ is constant. Similarly, show that the kinetic energy of rotation

$$T_{\rm rot} = \frac{1}{2}\lambda_1\omega_1^2 + \frac{1}{2}\lambda_2\omega_2^2 + \frac{1}{2}\lambda_3\omega_3^2$$

where the λ_i are the principle moments of inertia, is a constant of the motion.

(5) You are probably aware that the Earth's axis of rotation precesses slowly, so that, far in the future, the North pole will no longer be pointing at Polaris. To gain an understanding of this phenomenon, imagine that the Earth is perfectly rigid, uniform, and spherical and is spinning about its usual axis at its usual rate. A huge mountain of mass 10^{-8} Earth masses is now added at colatitude 60°, causing the earth to begin free precession as discussed in class. How long will it take the North Pole (defined as the northern end of the diameter along $\boldsymbol{\omega}$) to move 100 miles from its current position?