PHYS3101 Analytical Mechanics Homework #3 Due 19 Sep 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Two equal masses m are connected by a massless string of length L. The string passes through a hole in a frictionless horizontal flat table, so one mass slides freely on the table and the other hangs straight down. Use plane polar coordinates for the mass on the table, and show that the angular coordinate is ignorable. Find a solution to the equation of motion where the radial coordinate can be a constant, and then show that that coordinate is stable under small deviations from that constant.

(2) Consider a system of N different massive particles described by spherical polar coordinate (r, θ, ϕ). Assume that the system is symmetric under rotations about the z-axis. That is, a transformation from (r, θ, ϕ) to $(r, \theta, \phi + \epsilon)$ does not change the Lagrangian. (You can assume that there are no velocity-dependent potential energies.) Determine the associated conserved quantity, and interpret it physically.

(3) Show that the vector potential $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$ for a uniform, static magnetic field $\mathbf{B}$. Next express $\mathbf{A}$ in cylindrical polar coordinates (ρ, ϕ, z), including its direction with the appropriate unit vector(s). (Use the "smart choice" for the z-direction.) Now write down the Lagrangian and derive the equations of motion for a particle with mass m and charge q in this magnetic field. Describe the solutions of these equations when ρ is a constant. Recall Problem 2 from Homework 1.

(4) Write down the Lagrangian for a simple plane pendulum of length L and bob mass m using Cartesian coordinates (x, y) for the bob. Now write down a suitable constraint equation between x and y. (Many different choices are possible.) Use your constraint equation with a Lagrange multiplier to find modified Lagrange's equations, and show that the result is equivalent to using a single degree of freedom described by the angle θ through which the pendulum swings.

(5) A block of mass m slides down a frictionless inclined plane. Using the x and y coordinates shown, and the method of Lagrange multipliers, find the forces of constraint in the x and y directions. Show that this is just what you expect from your introductory physics course.