PHYS3101 Analytical Mechanics Homework #2 Due 12 Sep 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) A simple plane pendulum of length ℓ hangs from the ceiling of an elevator. The elevator is accelerating upwards at an acceleration a. (If $a < 0$, then the elevator is accelerating downwards.) Use the Lagrangian approach to find the frequency of small oscillations in terms of ℓ, a, and g. (This problem is a demonstration of Einstein's Principle of Equivalence.) It is probably easiest to write the Lagrangian first in terms of Cartesian coordinates.

(2) A mass M hang from a massless string that passes over a massless pulley. A similar pulley hangs from the other end of the string, over which a second massless string supports a mass m_1 on one side and a mass m_2 on the other side. Use Lagrange's equations to find the accelerations of M and m_1 when the system is released, in terms of m_1, m_2, M, and g. (If you find the necessary algebra at the end a little messy, I suggest you do it with MATHEMATICA.) Under what conditions is the acceleration of M equal to zero?

(3) A helix is determined in terms of its radius R and pitch λ using three dimensional spherical coordinates ρ, ϕ, and z as $\rho = R$ and $z = \lambda\phi$. If a particle of mass m is constrained to lie on the helix, which is oriented so that z is vertical, use Lagrange's equations to find the acceleration $\ddot{z}$ in terms of R, λ, and g. Discuss the behavior of $\ddot{z}$ for limiting values of R and λ, that is the cases $R \ll \lambda$ and $R \gg \lambda$.

(4) A particle of mass m is constrained to move without friction along a horizontal circular hoop of radius R. The hoop rotates with a fixed angular velocity ω about a fixed point on the circle. Show that the motion of the mass is the same as that for a vertical, plane pendulum, and find the frequency of small oscillations. Check that your answer is dimensionally correct. (In fact, if you think about the concept of a "centrifugal force", then you can check that you got the correct answer very simply.)

(5) Example 7.6 in Taylor derives the equation of motion (7.69) for the "bead on a spinning hoop" problem. Rewrite this equation in terms of dimensionless time $x \equiv \omega t$ and $\alpha \equiv g/\omega^2 R$. Assuming the bead starts from rest, numerically solve for $\theta(x)$ and plot it for several periods for each of the following cases:

(a) $\theta_0 = 0.1$ and $\alpha = 2$
(b) $\theta_0 = 1.1$ and $\alpha = 1/2$
(c) $\theta_0 = 0.1$ and $\alpha = 1/2$

Discuss the motion in each case, and compare to what you get from the linearized versions (7.72) or (7.80) in Taylor.