PHYS3101 Analytical Mechanics Homework #1 Due 5 Sep 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) A man of mass M straddles the gap between a train and the station platform, with his left foot on the train step and his right on the platform. He lifts his right foot at the same time as he jerks his bag with mass m onto the train. He pulls the bag with a constant horizontal acceleration and it lands on the train in a time T. If the distance between the bag and the train is L and the coefficient of static friction between his left foot and the train step is μ, derive a condition that can be tested to see if his foot slips and he falls into the gap. Use the average horizontal velocity of the bag to calculate the impulse on the man. If $M = 110$ kg, $L = 1$ m, $T = 1/4$ s, $m = 20$ kg, and $\mu = 0.4$, does the man fall into the gap?

(2) The force $\mathbf{F}$ on a particle with mass m and charge q moving with velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is given by $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. If $\mathbf{E} = E\hat{\mathbf{z}}$ and $\mathbf{B} = B\hat{\mathbf{z}}$, solve for the motion $\mathbf{r}(t)$ if the initial conditions are that the particle starts at the origin and

(a) $\mathbf{v}(t = 0) = 0$
(b) $\dot{x} = v_0$, and $\dot{y} = \dot{z} = 0$

(3) Use MATHEMATICA or some other application to reproduce the plot at the bottom of Figure 5.15 from Example 5.3 in the textbook. You are welcome to arrive at the solution with whatever tools you'd like. (I just used DSolve.) This example uses a damping coefficient $\beta = \omega_0/20$. Now repeat the calculation for $\beta = \omega_0/5$ and for $\beta = \omega_0/50$. Explain the physical origin of the differences and similarities of these three plots.

(4) Find an equation for the path $y = f(x)$ that makes the integral

$$S = \int_a^b x \left[1 - (f'(x))^2\right]^{1/2} dx$$

an extremum for fixed values of a and b.

(5) Consider a mass m moving in two dimensions (x, y) or (r, ϕ), subject to a potential energy function $U = kr^2/2$ where k is a constant. Write down the Lagrangian, find the equations of motion, and describe their solutions, when the particle's position is written in terms of

(a) coordinates x and y.
(b) coordinates r and ϕ.

For part (b), what is the significance of the solution for $\phi(t)$?