DART: Planetary Defense in the Introductory Physics Curriculum

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The Double Asteroid Redirection Test (DART) is a National Aeronautics and Space Administration–European Space Agency collaborative mission to test the feasibility of defending Earth from a catastrophic asteroid impact by using a spacecraft to deflect the asteroid away from the planet. Launched on Nov. 23, 2021, the DART spacecraft will intercept the binary asteroid 65803 Didymos in late September or early October 2022, colliding (nearly) head-on to modify its motion measurably.[1]

The DART mission would be an ideal supplement to the standard textbook treatment of a one-dimensional inelastic collision involving blocks sliding over a frictionless surface. It is certain to fascinate students and—in contrast to colliding blocks—vividly illustrate the contemporary, real-world importance of momentum conservation.

But the physics of the DART mission is even richer, addressing Kepler's laws, energy conservation, and orbital dynamics as well as momentum conservation, topics that are all included in a first-year mechanics course. DART could be an ideal end-of-semester review topic or the focus of a final examination question. In the following, we will describe the physics of DART in simplified form, as appropriate for either of these purposes. We first review the orbital motions of the spacecraft and the two bodies comprising the asteroid, and then calculate the change in the latter due to the collision. Finally, we expand the discussion beyond simple one-dimensional inelastic collisions to include those that are not head-on, or where a significant mass of matter is blasted off the asteroid surface after impact—as expected for DART.

The heliocentric orbits of DART and Didymos

As it approaches the asteroid, the DART spacecraft will be coasting, unpowered, on a near-circular orbit[2] (Fig. 1). Meanwhile, Didymos is following an elliptical orbit with eccentricity $\varepsilon = 0.384$ and semimajor axis $a = 1.644$ AU ($1 \text{ AU} = 1.496 \times 10^8$ km).[3] On Sept. 27, 2022, the estimated date of the intercept, the asteroid will be a distance $R = 1.043$ AU from the Sun.[4] To analyze the impending collision, we first need to know the speed of each body at this time.

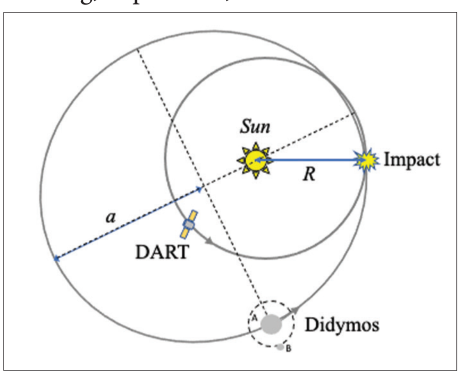

Fig. 1. The heliocentric orbits of DART and Didymos prior to impact. The semimajor axis of the asteroid trajectory a = 1.644 AU. DART follows a near-circular path, impacting the asteroid at a distance R = 1.043 AU from the Sun. The eccentricity of the spacecraft's orbit (<0.05) is greatly exaggerated in the figure.

The spacecraft speed v_s is found by equating its centripetal acceleration to the force per unit mass exerted by the Sun:

$$v_s^2/R = GM_\odot/R^2, \tag{1}$$

where $G = 6.674 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ is the gravitational constant and $M_\odot = 1.989 \times 10^{30}$ kg is the mass of the Sun. Calculations are simplified if the product $GM_\odot$ is expressed in more convenient units: $GM_\odot = 887\ \text{km}^2 \cdot \text{AU/s}^2$. Solving, $v_s = 29.2$ km/s.

The asteroid speed V is found using energy conservation: the sum of its kinetic and potential energies is constant throughout its orbit and equal to $E = -GM_\odot M/2a$, so

$$\frac{E}{M} = \frac{1}{2}V^2 - \frac{GM_\odot}{R} = -\frac{GM_\odot}{2a}, \tag{2}$$

where M is the asteroid's mass. Carrying out the calculation, $V = 34.1$ km/s; the asteroid is moving faster than the spacecraft, which also follows because $a > R$. If the *velocities* of the two bodies are parallel at the time of impact, the asteroid catches up to the spacecraft and "rear-ends" it with a relative speed of 4.9 km/s (11,000 mph). (One way to help students visualize such a violent, high-speed collision is to point out that just 1 s before impact, the two bodies are more than 3 miles apart.)

In the literature,[2] the relative speed is commonly estimated to be about 6.0 km/s. The difference is probably due to our simplifying assumption of a circular orbit for DART and our choice of the exact date of the encounter. For example, if the spacecraft is traveling on an elliptical orbit with semimajor axis 1.00 AU and eccentricity 0.05, its speed at impact would be 28.5 km/s on Sept. 27, so the relative velocity would be 5.6 km/s. In addition, from Sept. 25 to Oct. 5, 2022, the Sun–asteroid distance varies from 1.05 to 1.03 AU, while the asteroid's speed varies from 33.9 to 34.4 km/s.

The internal dynamics of Didymos

Didymos is a binary asteroid. The larger of its two bodies, called Didymos A, has an approximate diameter $d_A \cong 780$ m, while the approximate diameter of the smaller body (Didymos B or Dimorphos) is $d_B \cong 160$ m. The two components orbit their common center of mass (CM) with period $T = 11.92$ h and center-to-center distance $r_A + r_B = 1.18$ km, where r_A and r_B are the radii of their orbits [Fig. 2(a)].[5] For simplicity's sake, let us ignore the uncertainties in these numbers and also assume that both bodies are homogeneous spheres of equal density. With these assumptions and the above information, we can estimate the mass of each body.

The task is simplified by assuming circular orbits, which is consistent with observations. Referring to Fig. 2(a) and using

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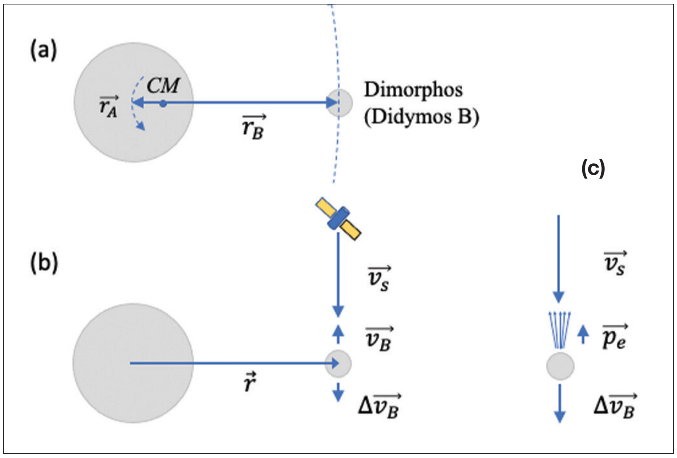

Fig. 2. (a) Didymos A and B rotate about their common center of mass, with a center-to-center distance of $r = r_A + r_B = 1.18$ km and period 11.92 h. (b) Since $m_A \gg m_B$, the CM lies very close to the center of A. The spacecraft approaches B with a relative speed $v_S \gg v_B$, causing a change Δv_B in the asteroid's speed. (c) The collision creates an impact crater, ejecting matter with momentum Δp_e, which amplifies the magnitude of Δv_B.

Newton's second law, the centripetal force on m_A is

$$\frac{m_A v_A^2}{r_A} = \frac{G m_A m_B}{(r_A + r_B)^2}, \qquad (3)$$

and similarly, for the centripetal force acting on m_B :

$$\frac{m_B v_B^2}{r_B} = \frac{G m_A m_B}{(r_A + r_B)^2}. \qquad (4)$$

But the orbital period is the same for both bodies: $T = 2\pi r_A/v_A = 2\pi r_B/v_B$, so, substituting for v_A and v_B in the above equations, we obtain

$$\frac{4\pi^2 r_A}{T^2} = \frac{G m_B}{(r_A + r_B)^2}$$

and (5)

$$\frac{4\pi^2 r_B}{T^2} = \frac{G m_A}{(r_A + r_B)^2}.$$

Adding the two equations, we obtain the generalized form of Kepler's third law:[6]

$$\frac{T^2}{(r_A + r_B)^3} = \frac{4\pi^2}{G(m_A + m_B)}, \qquad (6)$$

from which the total mass of the system can be calculated: $m_A + m_B = 5.28 \times 10^{11}$ kg. To estimate the mass of each body, use $m_B/m_A = (d_B/d_A)^3 = 0.00863$. Then, $m_B + m_A = 1.00863 m_A$, so $m_A = 5.24 \times 10^{11}$ kg and $m_B = 4.52 \times 10^9$ kg. Dimorphos (m_B), the smaller of the two bodies, is DART's target. See supplementary materials[7] for related student exercises.

How DART changes Dimorphos's orbit

Let us now view the collision in Didymos's center-of-mass reference frame [Fig. 2(b)]. Because $m_A \gg m_B$, we can safely ignore the larger body's motion and treat Dimorphos's trajectory as a circular orbit about a stationary m_A, with orbital radius $r_0 = r_A + r_B$. The orbital speed of Dimorphos is easily calculated using $v_B = 2\pi r_0/T$, with the result $v_B = 17.3$ cm/s. (This is five orders of magnitude smaller than the more familiar speeds of solar system planets. Hopefully, this will surprise and intrigue students as much as it did the author.)

The spacecraft's mass at the time of impact will be $m_s \cong 560$ kg.[1] If it undergoes a totally inelastic head-on collision with Dimorphos, as shown in Fig. 2(b), what will be the change of velocity Δv_B, and how will this affect its orbit?

In the following, assume that the spacecraft's velocity lies in the plane of Dimorphos's orbit. In fact, it may be inclined by as much as 17° from the orbital plane, but only the in-plane component of the velocity will have a significant effect on the motion.

Equating momenta before and after the collision,

$$m_s \mathbf{v}_s + m_B \mathbf{v}_B = (m_s + m_B)(\mathbf{v}_B + \Delta \mathbf{v}_B), \qquad (7)$$

where $\mathbf{v}_s$ is the velocity of the spacecraft relative to the asteroid, and $\Delta \mathbf{v}_B$ is the change in the asteroid's velocity due to the collision. Simplifying, with $m_s \ll m_B$, we obtain $\Delta \mathbf{v}_B \cong m_s \mathbf{v}_s/m_B$ or, since $\mathbf{v}_s$ is antiparallel to $\mathbf{v}_B$, $\Delta v_B = -m_s v_s/m_B = -0.6$ mm/s. Surprisingly, this tiny change in speed can be detected by astronomers millions of kilometers away, because it causes a small but measurable change in the orbital period of Dimorphos about the binary asteroid's CM.

The collision reduces Dimorphos's speed, and therefore its kinetic energy, but leaves its potential energy unchanged. This lowers its total energy by an amount

$$\Delta E = \tfrac{1}{2} m_B (v_B + \Delta v_B)^2 - \tfrac{1}{2} m_B v_B^2 \cong m_B v_B \Delta v_B \qquad (8)$$

and causes the asteroid to enter an elliptical orbit with semi-major axis $a' < r_0$. Using Eq. (2), the energies of the asteroid before and after the collision are $E = -G m_A m_B/2r_0$ and $E' = -G m_A m_B/2a'$, respectively, so

$$-\frac{G m_A}{2a'} = -\frac{G m_A}{2r_0} + v_B \Delta v_B. \qquad (9)$$

But for the initial circular orbit, $v_B^2 = G m_A/r_0$. Defining $\delta = \Delta v_B/v_B < 0$ we obtain

$$\frac{1}{a'} = \frac{1}{r_0}(1 - 2\delta), \qquad (10)$$

so $a' < r_0$. Moreover, since the velocity after the collision is still perpendicular to the position vector $\mathbf{r}$, the collision point coincides with the *apoapsis* of the resulting ellipse, the point where the distance between the two bodies is a maximum: $r_0 = (1 + \varepsilon)a'$. Solving for the eccentricity ε, we find $\varepsilon = -2\delta = 0.007$, and $a' = 1.17$ km.

This eccentricity and change in orbital size (10 m) are too small to be measured with high precision by Earth-bound astronomers. But the change in orbital period can be measured accurately, and indeed, this is the very reason that a binary asteroid was chosen for the DART mission. From Kepler's third law [Eq. (6)],

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{a'}{r_0}\right)^3, \tag{11}$$

where T and T' are the orbital periods before and after the collision. Setting $T' = T + \Delta T$ and expanding, we obtain $1 + 2\,\Delta T/T = (a'/r_0)^3$, or

$$\Delta T = \frac{T}{2}[(a'/r_0)^3 - 1]. \tag{12}$$

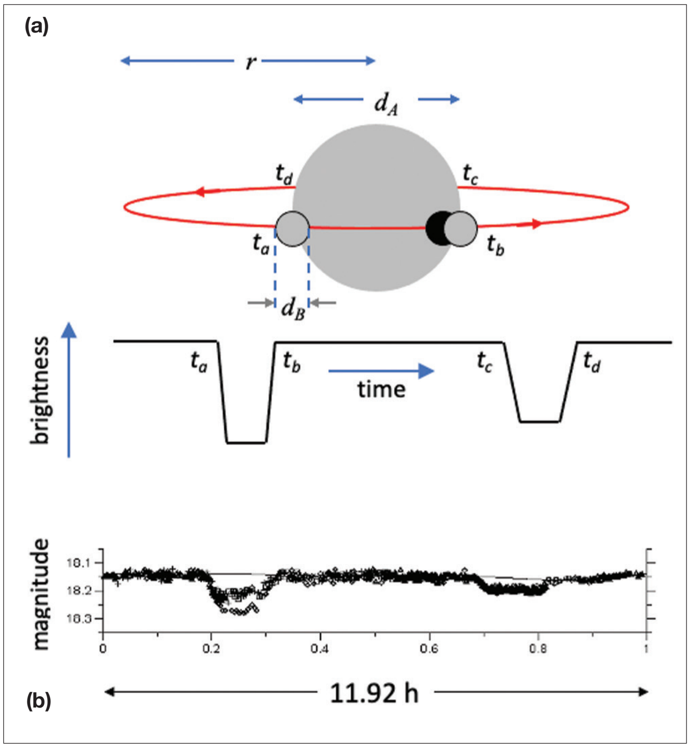

Fig. 3. (a) When Dimorphos (Didymos B) passes in front of or behind Didymos A, as viewed from Earth, the binary asteroid's total brightness appears to dim because one body blocks all or part of the reflected light from the other. In the former case, called a primary occultation, the shadow cast by B on A causes an additional dip in brightness. (b) Observed light curve from Didymos, adapted from Ref. 3.

Using the values of T and a'/r_0 found above, $\Delta T \approx 3\delta = -7.5$ min.

How "observable" is this change in period? Each full orbit of Dimorphos will be shorter by 7.5 min from what it was prior to the collision. This difference accumulates with each additional orbit: two full orbits will be completed 15 min faster than before, three orbits 22.5 min faster, and so on. Astronomers can measure the orbital period by observing the light curve from the double asteroid: each time Dimorphos passes in front of or behind its larger companion (as viewed from Earth), the total reflected light from the asteroid will dip slightly.[3] As illustrated by Fig. 3, the time duration of a transit is $\Delta t \approx (d_A + d_B)/v_B = 1.7$ h. Therefore, it will take about 14 orbits, or 7 d, to accumulate a time delay comparable to Δt. This will be easily observable from Earth.

Crater formation and the enhancement factor

When DART collides with Dimorphos, it will excavate an impact crater and spew ejecta outward from the asteroid's surface [Fig. 2(c)]. Let $\boldsymbol{p}_e$ be the momentum of the ejected material. Equating the momenta before and after the collision,

$$m_s\boldsymbol{v}_s + m_B\boldsymbol{v}_B = (m_s + m_B)(\boldsymbol{v}_B + \Delta\boldsymbol{v}_B) + \boldsymbol{p}_e, \tag{13}$$

or, ignoring second-order terms,

$$m_s\boldsymbol{v}_s = m_B\,\Delta\boldsymbol{v}_B + \boldsymbol{p}_e. \tag{14}$$

In the literature, it is customary to define the momentum enhancement factor β by the following expression: $\boldsymbol{p}_e = -m_s(\beta - 1)\boldsymbol{v}_s$, where the leading negative sign arises because the material is ejected backwards (in the $-\boldsymbol{v}_s$ direction). Solving, $m_B\Delta v_B = \beta m_s v_s$. In other words, if $\beta = 1$, no material is ejected, and if $\beta = 2$, the ejecta momentum has the same magnitude as the spacecraft momentum. The value of β depends critically on the strength and porosity of the asteroid's surface material. In simulations of high-speed collisions, β ranges from about 1.1 for surfaces with high cohesive strength to greater than 5 for surfaces with low cohesion and low porosity.[8] For an asteroid characterized as a "rubble pile" (low cohesion and large-scale porosity), such as the asteroid Itokawa, which was visited by the spacecraft Hayabusa in 2005, $\beta \approx 2$. Since Didymos has a density similar to Itokawa's, all of the effects on the orbital motion of Dimorphos are likely to be doubled from the values calculated in the previous section.

Collisions that are not head-on

What would happen to the orbit of Dimorphos if the collision were not head-on? While the general case is rather complicated,[9] the special case $\boldsymbol{v}_s \perp \boldsymbol{v}_B$ can be solved by appealing to the properties of an ellipse. The target asteroid's orbit after the collision is again an ellipse, but with a smaller eccentricity $\varepsilon = \delta$. To first order in δ, the semimajor axis is unchanged, i.e., $a' \cong r_0$, so that the orbital period is also hardly changed. See the Appendix[10] for a full analysis. Clearly, the ideal alignment for DART is with $\boldsymbol{v}_s \parallel \boldsymbol{v}_B$.

Conclusion

The DART mission is an ideal way for instructors to "wrap up" an introductory mechanics course. A concise analysis of the mission exploits many of the key concepts presented in the course, such as relative velocity, circular motion, universal gravitation, Kepler's laws, and the conservation of energy and momentum. Given the popularity of the recent movie "Don't Look Up," DART is sure to capture the imagination of students while illustrating the vital importance of classical mechanics in the contemporary world, beyond the classroom.

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10. Readers can access the Appendix at https://doi.org/10.1119/5.0085522, under the Supplemental tab.

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