Physics with Mathematica Fall 2019 Exercise #5 25 Sep 2019

Orbits in an inverse power law central potential

Set up the differential equations for motion in the xy-plane under an attractive central force $f = k/r^\beta$, and initial conditions $x(0) = a$, $y(0) = 0$, $\dot{x}(0) = 0$, and $\dot{y}(0) = v_0$. Pick some values for m, k, and a that you will use to make some plots.

First solve the equations for $\beta = 2$, and $v_0 = v_{\text{Circ}}$, the speed you calculate for a circular orbit. Use "ParametricPlot" to convince yourself the orbit is circular. Execute the solution for at least two circular orbit periods to convince yourself that the orbit is in fact closed.

Next, use v_0 below and above the circular values, and convince yourself that you see closed elliptical orbits, or, if you choose a large enough v_0, open hyperbolic or parabolic orbits.

Then choose $\beta = 3$ and again check for closed circular orbits when $v_0 = v_{\text{Circ}}$. Investigate what happens when you move away from the initial circular velocity.

Send the grader an email with your notebook as an attachment.