Physics with Mathematica Fall 2019 Exercise #4 18 Sep 2019

Motion of a Damped Forced Linear Oscillator

The equation of motion for a damped forced oscillator is

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t$$

It is convenient to express the motion in terms of the natural period $\tau_0 \equiv 2\pi/\omega_0$ and the driving period $\tau \equiv 2\pi/\omega$. Solve for the motion and plot $x(t)$ including the initial conditions which shows the transient behavior. This is avoided in classes because the math is onerous, but it's easy with MATHEMATICA.

a. Solve and plot for $0 \le t \le 10$, using $\beta = 0.1$, and natural period $\tau_0 = 1$, subject to the initial conditions $x(0) = \dot{x}(0) = 0$. Try first for a driving period $\tau = 2$ and amplitude $A = 1$. Then, try other choices. Consider the cases $\beta < \omega_0$ and $\beta < \omega_0$.

b. Use manipulate to study the behavior of the forced oscillator. Probably the most dramatic parameter to manipulate is τ, and let it span over τ_0 so that you can observe resonance.

Send the grader an email with your notebook as an attachment.