

DISTRIBUTION OF DARK MATTER IN THE SPIRAL GALAXY NGC 3198

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ABSTRACT

Two-component mass models, consisting of an exponential disk and a spherical halo, are constructed to fit a newly determined rotation curve of NGC 3198 that extends to 11 disk scale lengths. The amount of dark matter inside the last point of the rotation curve, at 30 kpc, is at least 4 times larger than the amount of visible matter, with $(M/L_B)_{\text{tot}} = 18 M_{\odot}/L_{B\odot}$. The maximum mass-to-light ratio for the disk is $M/L_B = 3.6$. The available data cannot discriminate between disk models with low M/L and high M/L , but we present arguments which suggest that the true mass-to-light ratio of the disk is close to the maximum computed value. The core radius of the distribution of dark matter is found to satisfy $1.7 < R_{\text{core}} < 12.5$ kpc.

Subject headings: galaxies: individual — galaxies: internal motions — interstellar: matter

I. INTRODUCTION

The problem of dark matter surrounding spiral galaxies, made evident by the flatness of rotation curves, is one of the most enigmatic questions in present-day astrophysics. A number of years of intensive research have brought little or no clarification, and suggestions offered in explanation of flat rotation curves include extremes like the possibility that Newtonian dynamics is at fault (Milgrom 1983; Sanders 1984) and Kalnajs's (1983) conclusion that—for some data sets—there is no problem at all.

In order to set the stage for the discussion in this paper, and to elucidate the confusing situation noted above, it is useful to review what one expects for the shape of the rotation curve. Let us assume that a typical spiral galaxy consists of two distinct distributions of matter: an exponential disk and a de Vaucouleurs spheroid, each with constant M/L . Surface brightness distributions of about half of the galaxies surveyed so far can indeed be explained by the sum of two such components (Borson 1981; Wevers 1984). Following Borson, we further assume that deviations from this simple picture shown by other galaxies can be attributed to irregularities in the distribution of bright young stars contributing little to the total mass. The maxima of the rotation curves for spheroid and disk lie at 0.3 effective radii and at 2.2 disk scale lengths, respectively. In the interval between these two points the shape of the rotation curve depends on the ratio of spheroid to disk mass, but it can be quite flat. Beyond about three disk scale lengths the rotation curve will show an approximately Keplerian decline.

In comparing this model with observations we first consider optical data. Rubin *et al.*'s rotation curves typically extend to 0.8 times the radius, R_{25} , of the 25th mag arcsec⁻² isophote (Rubin 1983). Since the central surface brightnesses of disks of spiral galaxies lie in the range $20.5 < B_c(0) < 23.0$ (Borson 1981; Wevers 1984), optical velocity information stops at 1.5 to 3.5 disk scale lengths. At 3.5 scale lengths the rotation curve of an exponential disk has decreased only 8% cent relative to the maximum value (and even less for a truncated disk; Casertano 1983). With optical data alone, it is not easy to see the Keplerian decline of the rotation curve. In many cases, a combined model with a bulge and a disk can produce a circular

velocity that is nearly independent of radius, without the need for nonluminous matter.

Another important aspect of this problem is that Rubin *et al.* (1982; see also Rubin 1983) find a well-established change in shape of the rotation curve with luminosity for Sb galaxies. High-luminosity Sb galaxies show a rapid rise of rotation velocity, V_{cir} , with fractional radius and then reach a more or less constant level, while rotation curves of low-luminosity Sb galaxies show a more gradual rise. [Note that the scaling of radius in terms of R_{25} corresponds to expressing the radius in the number of disk scale lengths if there is no scatter in $B_c(0)$.] On the bulge disk picture, this progression in shape with luminosity could only be explained if there exists a strong correlation of bulge-to-disk ratio with luminosity *within* a given Hubble type. The sense required is that high-luminosity galaxies have a prominent bulge—in terms of mass—while low-luminosity galaxies have no bulge at all. We show for illustrative purposes in Figure 1 the rotation curve for a high-luminosity galaxy, with spheroid and disk parameters adjusted in such a way that the rotation curve is flat for a large range in radii. Rotation curves *and* surface photometry are necessary to test whether bulge and disk properties deduced from the photometry are consistent with the assumption that such a bulge and disk, with constant M/L , produce the rotation curve observed.

We conclude from the above discussion that one can explain the shape of rotation curves inside three disk scale lengths, at least in principle, by a combination of matter distributions with constant M/L , without an additional component of dark matter. In this respect our conclusions agree with those of Kalnajs (1983). However, rotation curves obtained from the 21 cm line of neutral hydrogen gas in the outer region of spiral galaxies leave no doubt that "dark halos" do exist³ (Faber and Gallagher 1979; Bosma 1981). Thus one must have both optical (for the inner regions) and H I data (for the outer regions) to obtain a complete picture of the distribution of matter.

³ As pointed out by Freeman (1970) in an epochal paper, rotational velocities of H I (for NGC 300) do not show the expected decline at large radii (that is, beyond the turnover point of the disk). Freeman concluded: "If the H I rotation curve is correct, then there must be undetected matter beyond the optical extent of NGC 300; its mass must be at least of the same order as the mass of the detected galaxy."

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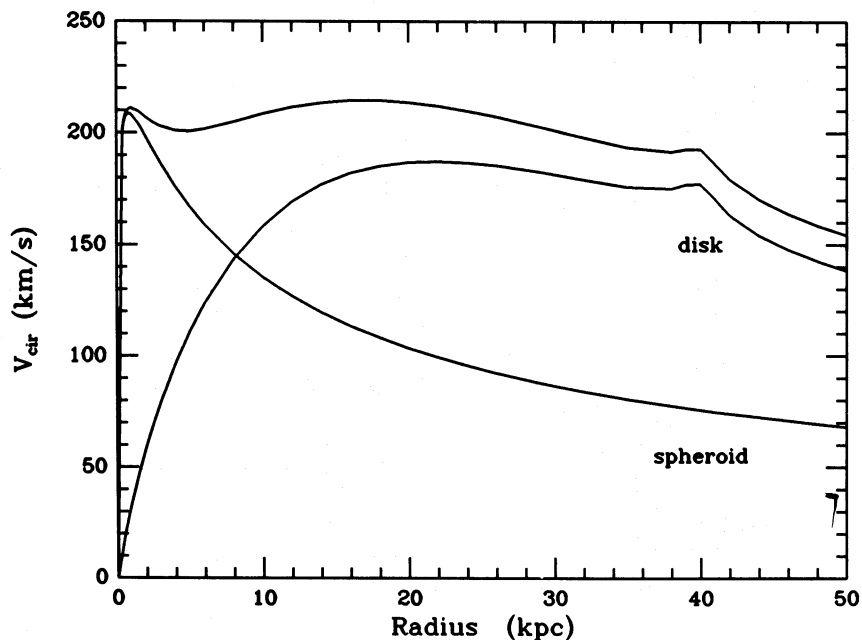


FIG. 1.—Rotation curve due to the sum of a de Vaucouleurs spheroid and a truncated exponential disk. The parameters have been chosen such that the rotation curve is flat over a large range in radii. The peak of the rotation curve of the spheroid lies at 0.8 kpc. Rotation curves such as these can probably explain observed flat rotation curves inside the luminous body of a galaxy, without the need for an additional component of dark matter. $R_{\text{eff}}(\text{spheroid}) = 2.7$ kpc; scale length disk, $h = 10$ kpc; truncation radius disk, $R_{\text{trunc}} = 4h$; mass ratio $M_{\text{sph}}/M_{\text{disk}} = 0.25$.

Unfortunately, the distribution of neutral hydrogen in the outer regions of galaxies is sometimes irregular, and, in galaxies seen edge-on, it is not always clear whether H I gas is present at the line of nodes. In such cases, a precise measurement of the circular velocity is not possible. But there are also several galaxies with warped H I disks which can be represented successfully by a tilted ring model. For moderate warps such a tilted ring model should be good enough to allow a reliable determination of the circular velocity.

These considerations show that galaxies with large, relatively unperturbed hydrogen disks, seen at inclinations of say 50° to 80° , are required for studies of the distribution of dark matter. A good example meeting these criteria is NGC 3198. This Sc galaxy, which has no nearby bright companions, has been observed by Bosma (1981) with the WSRT. Its velocity field is regular and agrees with that for a disk in differential rotation. In addition, the H I distribution extends to at least $2.3R_{25}$, even though Bosma's observations were not particularly sensitive. New 21 cm line observations with the WSRT, with improved sensitivity, have recently been obtained by one of us (Begeman, in preparation). The rms noise in these new H I maps is a factor of 4 smaller than in Bosma's maps, and the velocity field of H I can now be determined out to $2.7R_{25}$ (1.9 Holmberg radii), which corresponds to 11 scale lengths of the disk (see below). Even at these large distances from the center, deviations from axial symmetry in the velocity field are small. To our knowledge, this is the largest number of disk scale lengths over which a galactic rotation curve has been measured. A map of H I contours superposed on a IIIa-J photograph is shown in Figure 2 (Plate 13).

In this paper we discuss the implications of these observations, in combination with photometry, for the distribution of matter in NGC 3198. Surface photometry of NGC 3198 (Wevers 1984) shows that a single component, i.e., an exponen-

tial disk with a scale length of $60''$, gives an adequate representation of the distribution of light. Since the circular velocity is essentially constant beyond 2.5 disk scale lengths, this implies that a mass distribution with constant M/L is ruled out.

We find that the amount of dark matter associated with NGC 3198 inside 30 kpc, that is, the outermost point of the rotation curve, is at least 4 times larger than the visible mass. Inside one Holmberg radius (15.9 kpc) the ratio of halo mass to maximum disk mass is 1.5.

The organization of the paper is as follows. In § II and III we describe the distribution of light and the rotation curve. Two-component mass models fitted to the observed rotation curve are presented in § IV; the results are discussed in § V.

II. DISTRIBUTION OF LIGHT

Surface photometry of NGC 3198 in three colors (U' , $\lambda \approx 3760 \text{ \AA}$; J , $\lambda \approx 4700 \text{ \AA}$; F , $\lambda \approx 6400 \text{ \AA}$) has been published by Wevers (1984). An earlier study of van der Kruit (1979) gives photometry in the J band. Although the photograph of NGC 3198 in Bosma (1981) shows a distinct nucleus, Wevers's radial luminosity profiles can in first approximation be fitted with an exponential disk. Fitting straight lines to these profiles by eye, we find the following scale lengths: U' , $63''$; J , $58''$; F , $54''$, with uncertainties of 5%.

Comparison of the U' and F profiles shows that, near the center, there is a distinct color gradient, with the central region being redder. This may be related to the presence of a bulge or a depletion of young stars. For the purposes of this paper, an exponential law is a satisfactory representation of the light distribution; we adopt a scale length of $60''$, corresponding to 2.68 kpc for $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. (From the rotation curve we also find that the bulge must be small; see § IV). A summary of properties of NGC 3198 relevant for this paper is given in Table 1.

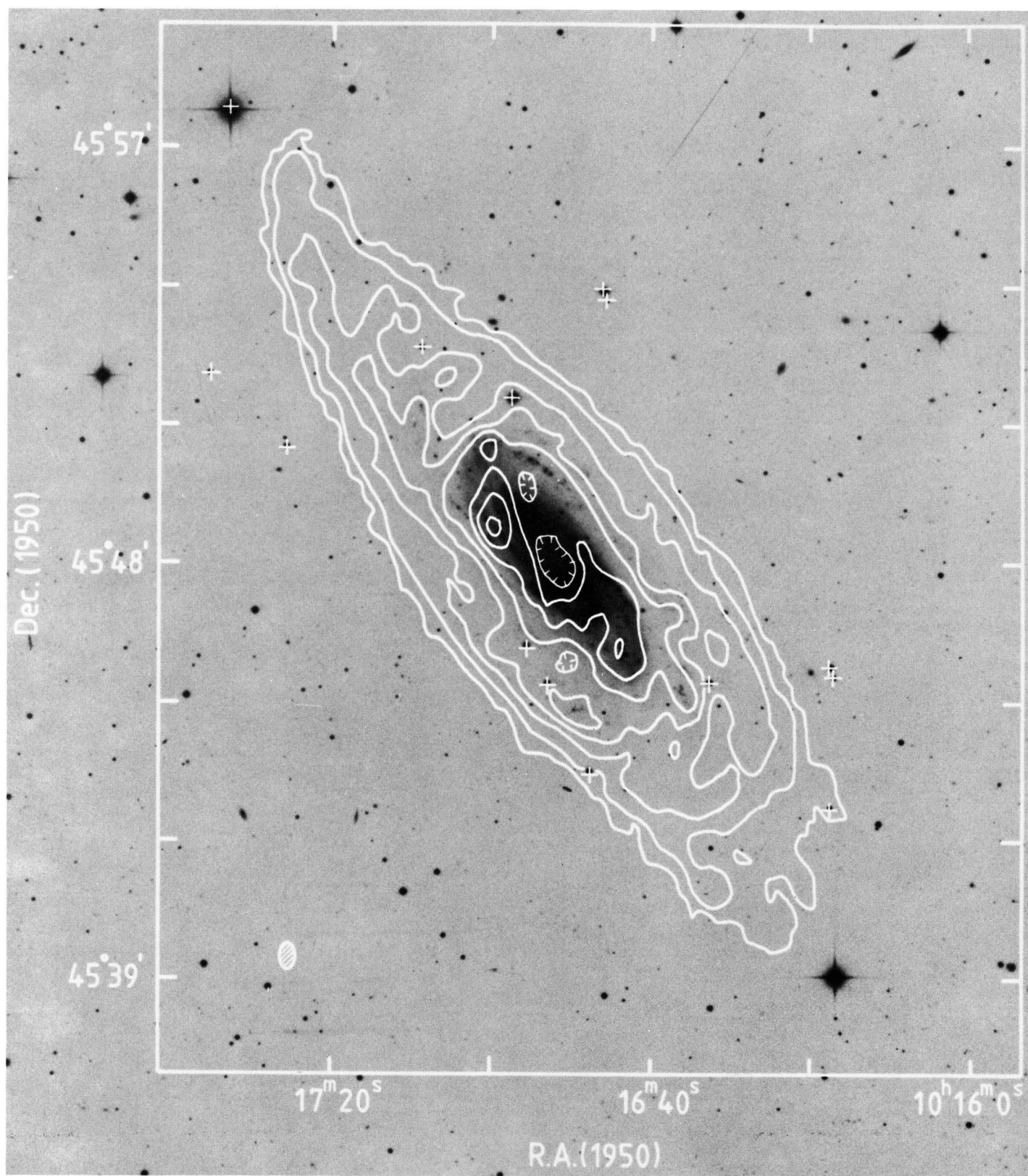


FIG. 2.—Full resolution ($25'' \times 35''$) map of the H I column density distribution of NGC 3198 superposed on an optical image of the galaxy (IIIa-J) (with Wr 2c), kindly made available by J. W. Sulentic. Contour levels are 1, 4, 8, 12, ..., 28 times 10^{20} atoms cm^{-2} .

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TABLE 1
PROPERTIES OF NGC 3198

Parameter	Value	Notes
Type	Sc(rs)I-II	1
V_{hel} (km s ⁻¹)	660 ± 1	2
Distance (Mpc)	9.2	3
B_T^0	10.45	4
M_{BT}^0	-19.36	
$(B-V)_T^0$	0.42	4
$L_B/L_{B\odot}$	$(8.6 \pm 0.9) \times 10^9$	
$L_V/L_{V\odot}$	$(7.0 \pm 0.7) \times 10^9$	
Scalelength disk	60" (= 2.68 kpc)	
R_{25}	4.2	4
Holmberg diameter ...	11.9 × 4.9	5

NOTES.—(1) Sandage and Tammann 1981. (2) Begeman 1985. (3) $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. (4) Radius 25th mag arcsec⁻² isophote; de Vaucouleurs, de Vaucouleurs, and Corwin 1976. (5) Holmberg 1958.

III. ROTATION CURVE

A rotation curve was derived from the new WSRT observations (FWHM beam 30") as follows. (Full details are given elsewhere; Begeman, in preparation). We represent the hydrogen disk by a number of circular rings, each ring being characterized by an inclination i , a position angle ϕ , and a circular velocity V_{cir} . The width of the rings is 30" on the major axis. Excluding a sector with opening angle 90° (in the plane of the galaxy) about the minor axis, we use the grid points at which the velocity field is recorded inside each ring to obtain a least-squares solution for i , ϕ , and V_{cir} as a function of radius. An advantage of this method is that ring-to-ring variations in i and ϕ , in combination with the formal errors from the least-squares solution, indicate to what extent the hydrogen disk is warped. As expected, the position angle is well determined, while V_{cir} and i are not completely independent. Yet, taking the dependence into account, the formal errors in V_{cir} and i are extremely small: 1 km s⁻¹ and 1°, respectively. We find small variations of inclination with radius: from 72° inside 2', through a

minimum of 70° at 6', to 76° at 10' from the center. There is also a systematic variation of a few degrees in position angle with radius. Thus there appears to be a small warp of the disk; it corresponds to a vertical displacement of 2.3 kpc at the "edge" (29 kpc from the center).

As a further check of these findings, the same procedure was applied to the northern and southern halves of the galaxy separately. The results clearly show the symmetric large-scale structure of NGC 3198: both halves show the same dependence of position angle and inclination on radius, and in the region beyond 6' from the center, i.e., the interval that is most critical for the subject of this paper, the inclinations derived separately for the two halves agree to within 1° (see Fig. 3). On the other hand, the rotation curves for the two halves are slightly different: in the southern half there is almost no change in V_{cir} with radius beyond the maximum at 3', but in the northern half V_{cir} decreases slowly between 3' and 8' by a few km s⁻¹ and then rises between 8' and 11'. The maximum difference between the two halves is 6 km s⁻¹.

In the inner region, i.e., inside ~3', this method of deriving the circular velocity does not work since there are only a few grid points per ring. Moreover, the gradient in the velocity field across a ring, and across the beam, must be taken into account. As described by Begeman, for small radii the circular velocity can be derived from an l - v diagram along an adopted direction for the major axis and with an adopted value for the inclination of the disk (l is the position along the major axis). Using the information in the l - v diagram a first estimate of the rotation curve is obtained by plotting the velocities corresponding to peak intensity, corrected for inclination, against l . A correction for beam smearing is then calculated by taking a model velocity field and convolving it with the WSRT beam. (A correction for beam smearing was calculated and applied for all radii, but beyond 3' this correction becomes negligibly small.) The rotation curve determined by Cheriguene (1975) from the motion of H II regions 2' from the center agrees well with the l - v diagram of H I at the position angle of her data for $r \geq 60$ ". At

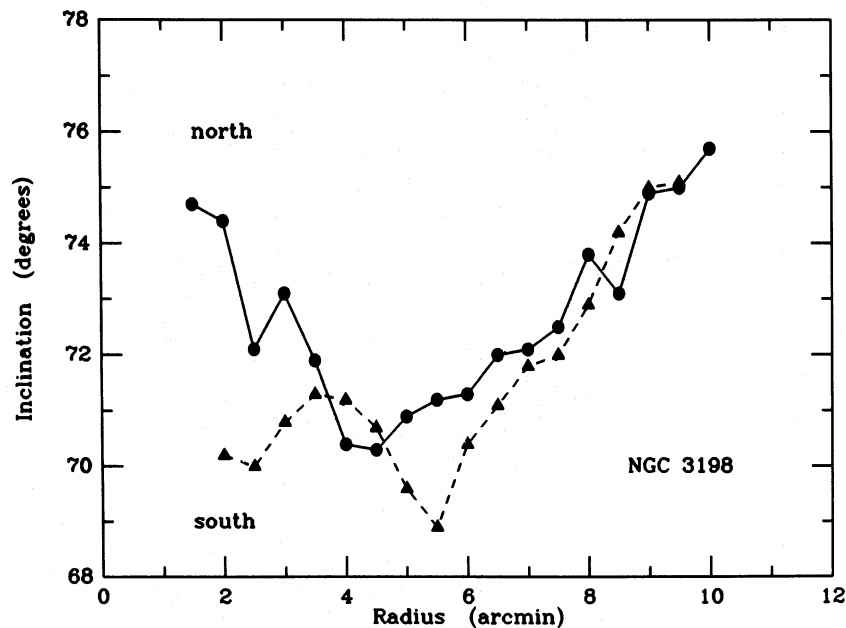


FIG. 3.—Variation of inclination angle with radius in NGC 3198, according to a tilted ring fitted to the velocity field, separately for the northern and southern half of the galaxy. 1' corresponds to 2.68 kpc.

TABLE 2
H I ROTATION CURVE OF NGC 3198

DISTANCE FROM CENTER	V_{hel} (km s ⁻¹)	DISTANCE FROM CENTER	V_{hel} (km s ⁻¹)
0.25	55 ± 8	4.5	153 ± 2
0.50	92 ± 8	5.0	154 ± 2
0.75	110 ± 6	5.5	153 ± 2
1.00	123 ± 5	6.0	150 ± 2
1.25	134 ± 4	6.5	149 ± 2
1.50	142 ± 4	7.0	148 ± 2
1.75	145 ± 3	7.5	146 ± 2
2.00	147 ± 3	8.0	147 ± 2
2.25	148 ± 3	8.5	148 ± 2
2.50	152 ± 2	9.0	148 ± 2
2.75	155 ± 2	9.5	149 ± 2
3.00	156 ± 2	10.0	150 ± 2
3.50	157 ± 2	10.5	150 ± 3
4.00	153 ± 2	11.0	149 ± 3

30" from the center, the beam-corrected H I rotation velocities are ~ 20 km s⁻¹ higher than those of Cheriquene. H I emission on the scale of the beam (30") has been detected everywhere in the central region.

The final rotation curve is given in Table 2. Inside 2.5 they correspond to 0.5 times the mean difference between the rotation curves derived separately for the northern and southern halves (~ 2 km s⁻¹). The last two points of the rotation curve are based on adopted values for position angle and inclination.

Between 5' and 9', Bosma's (1981) rotation velocities are systematically lower than the new ones by ~ 7 km s⁻¹. This is partly due to the use of a fixed inclination as a function of radius by Bosma. The cause of the remaining difference is not entirely clear. The sense of Bosma's velocity residuals is such that a somewhat larger rotation velocity in the region of interest would have improved the fit. Due to the correction for beamsmeearing—amounting to 23 km s⁻¹ at 30" from the center—the new rotation curve rises more steeply than Bosma's.

IV. MASS MODELS

a) Choice of Components

We consider mass models consisting of the following two components: (i) a thin exponential disk (de Vaucouleurs 1959; Freeman 1970); (ii) a spherical halo, representing the distribution of dark matter:

$$\rho_{\text{halo}}(R) \propto \left[\left(\frac{a}{R_0} \right)^\gamma + \left(\frac{R}{R_0} \right)^\gamma \right]^{-1}, \quad (1)$$

where R_0 is a fiducial radius (Bahcall, Schmidt, and Soneira 1982; hereafter BSS). The parameter a , which is linked to the core radius, and the exponent γ can be varied freely. We choose R_0 equal to 8 kpc. This allows a comparison of $\rho_{\text{halo}}(R_0)$ for the models with the halo mass density in the solar neighborhood (~ 0.01 – $0.02 M_\odot \text{ pc}^{-3}$; Bahcall and Soneira 1980). Equation (1) is equivalent to

$$\rho_{\text{halo}}(R) = \rho_{\text{halo}}(0) \left[1 + \left(\frac{R}{a} \right)^\gamma \right]^{-1}.$$

In addition, we estimate the maximum mass of a bulge component, represented by a de Vaucouleurs spheroid, from the shape of the rotation curve in the inner region. To model the

spheroid we use the approximations given by Young (1976; see BSS, Table 1).

b) Fits with Exponential Disk and Halo

To limit the number of free parameters of the models we shall only consider exponential disks with scale lengths equal to that of the light distribution.⁴ This is equivalent to the use of exponential disks with M/L independent of radius. As described in § II, there is no clear evidence for a bulge in NGC 3198. Therefore we will first restrict ourselves to the combination of a disk and halo. The distribution of light also shows that any truncation of the disk must occur outside 5.5 scale lengths. For such a large truncation radius the difference between a truncated and an untruncated disk is very small as far as the rotation curve is concerned (Casertano 1983; Bahcall 1983). It will therefore be sufficient to consider only the simple case of infinite disks. Another reason for considering infinite disks only is that in the outer regions the contribution of H I to the total mass density cannot be neglected. Between 5.5 and 11' the surface density of H I is well represented by an exponential with scale length 3.3. For the maximum disk case to be discussed below the surface densities of stars and H I are about equal at $R = 5.5$. The total amount of H I present in NGC 3198 is $4.8 \times 10^9 M_\odot$; its contribution to the maximum disk mass is 15%.

Our first model consists of a disk with the largest possible mass and a halo. A strict upper limit for V_{max} of this disk is 150 km s⁻¹. This choice would require a halo with a hollow core, however, which is implausible. Thus V_{max} must be somewhat smaller. We find that a reduction of $V_{\text{max}}(\text{disk})$ to 140 km s⁻¹ is sufficient to allow a halo with a density that decreases monotonically with galactocentric distance. The fit of this disk (total mass $3.1 \times 10^{10} M_\odot$) and halo to the observed rotation curve is shown in Figure 4. The parameters deduced for the halo are not unique: curves with two free parameters are generally sufficient for mass modeling purposes (Kormendy 1982). In this case the halo exponent γ and the scale length a can be varied in a correlated fashion ($1.9 < \gamma < 2.9$, $7 < a < 12$), while $\rho(R_0)$ is fixed to within a narrow range. This freedom in the choice of halo parameters need not concern us: our main interest is the mass distribution in the dark halo, which follows directly from the shape of the rotation curve for the halo component. The latter is fixed by the observed rotation curve and the adopted disk. Defining the core radius of the halo mass distribution with $\rho(R_{\text{core}}) = 2^{-3/2} \rho(R = 0)$, we have $R_{\text{core}} = (2^{3/2} - 1)^{1/\gamma} a$. From this we find the—subjective—95% confidence interval $9.6 < R_{\text{core}} < 15.4$. Thus, $R_{\text{core}} = 12.5 \pm 1.5$ (1 σ) kpc = 4.7 ± 0.6 disk scale lengths. Note that this is an upper limit for R_{core} : for smaller disk masses R_{core} decreases (see below). Cumulative mass distributions for disk and halo are shown in Figure 5. From this figure it follows that the ratio of

⁴ Following a suggestion by J. P. Ostriker and the referee, we have checked the exponential disk approximation of the light distribution by calculating rotation curves directly from the light profiles (assuming an infinitely thin disk). The rate of decline beyond the maximum agrees well with that for an exponential disk with scale length 60" for all three curves (U' , J , and F). In the inner region there is also good agreement between the exponential disk rotation curve and the rotation curve calculated from the F profile, which presumably gives the better indication of the underlying mass distribution. The U' rotation curve rises less steeply, indicating that compared to F , the dominant population contributing to U' has a central depression in its radial distribution. No J data are available in the inner region. These results confirm our choice of 60" for the scale length of the disk. We thank S. Casertano for lending us his programs to perform these calculations.

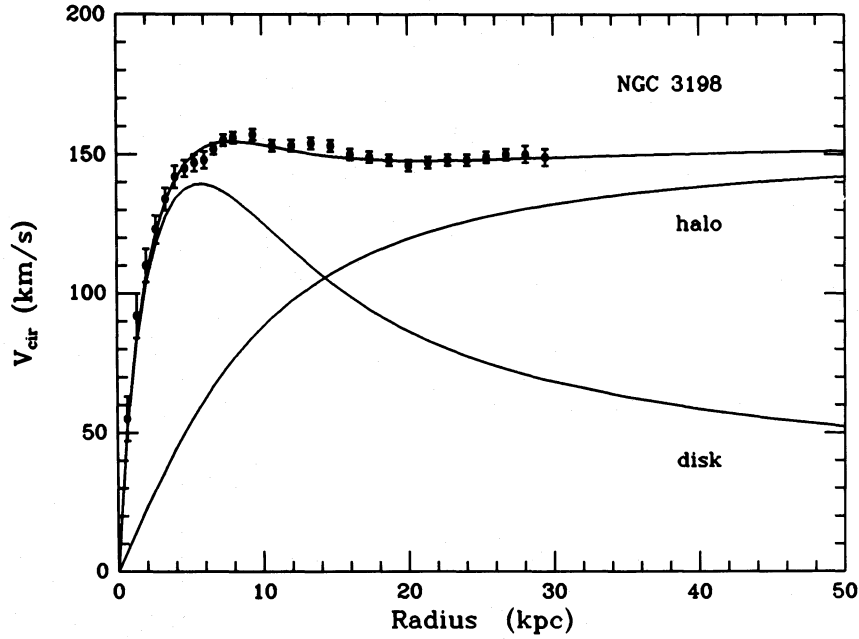


FIG. 4.—Fit of exponential disk with maximum mass and halo to observed rotation curve (dots with error bars). The scale length of the disk has been taken equal to that of the light distribution ($60''$, corresponding to 2.68 kpc). The halo curve is based on eq. (1), $a = 8.5$ kpc, $\gamma = 2.1$, $\rho(R_0) = 0.0040 M_\odot \text{pc}^{-3}$.

dark matter to visible matter inside the last point of the rotation curve (at 30 kpc) is 3.9. The enclosed halo mass is 0.8 times the disk mass at R_{25} ; the enclosed halo mass is 1.5 times the disk mass at the Holmberg radius. The total mass inside 30 kpc is $15 \times 10^{10} M_\odot$. Another property of interest is the mass-to-light ratio of the disk; we find $M/L_B(\text{disk}) \leq 3.6 M_\odot/L_{B_\odot}$ and $M/L_V(\text{disk}) \leq 4.4 M_\odot/L_{V_\odot}$.

The disk-halo model shown in Figure 4 has the characteristic flat rotation curve over a large part of the galaxy. Beyond 30 kpc it is a mere extrapolation, but the observations inside 30

kpc do not show any sign of a decline, and the extrapolated curve may well be close to the true one. To obtain an estimate of the minimum amount of dark matter at large distances from the center we have also made a fit, shown in Figure 6, with a halo density law whose slope changes from -2 in the inner region to -4 in the outer region:

$$\rho_{\text{halo}}(R) \propto \left[\left(\frac{a}{R_0} \right)^2 + \left(\frac{R}{R_0} \right)^2 + 0.08 \left(\frac{R}{R_0} \right)^4 \right]^{-1}, \quad (2)$$

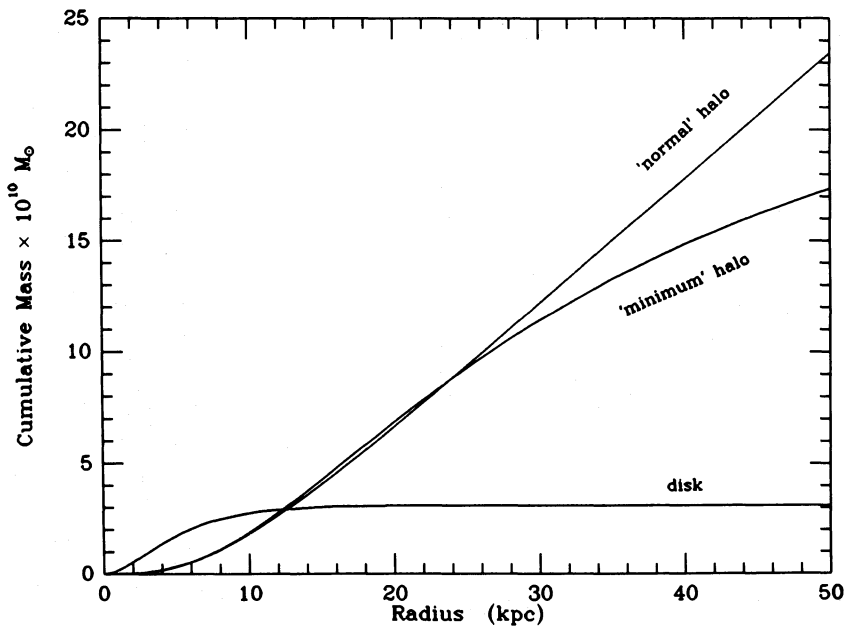


FIG. 5.—Cumulative distribution of mass with radius for disk and halo for the maximum disk mass case. Two halo fits are shown. The curve labeled “normal” halo is based on eq. (1); the parameters of the fit are the same as those in Fig. 4. The curve labeled “minimum” halo is based on eq. (2); it corresponds to a density distribution whose slope changes from -2 in the inner regions to -3.5 in the outer regions. This curve represents an estimate of the minimum amount of dark matter in NGC 3198 inside 50 kpc.

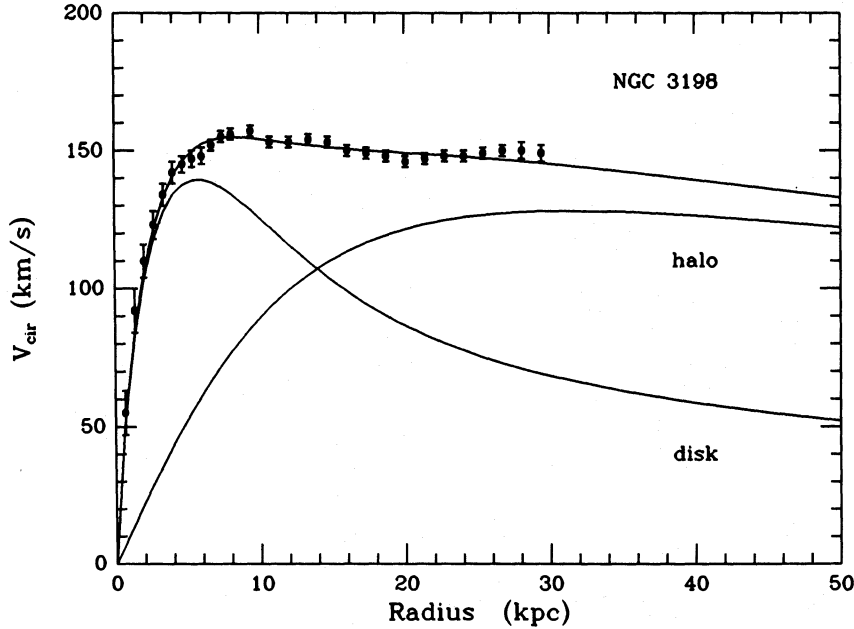


FIG. 6.—Fit of “maximum” disk and “minimum” halo to observed rotation curve. The halo is represented by eq. (2). See also Fig. 4.

where $\rho_{\text{halo}}(R_0) = 0.0042 M_{\odot} \text{pc}^{-3}$, $a = 10$ kpc, and $R_0 = 8$ kpc; see BSS. Gradients for this density law are $d \log \rho / d \log R = -2, -3, -3.5$ at $R = 17, 32,$ and 48 kpc, respectively. (These values, we feel, are not unreasonable. One could, of course, also simply truncate the halo at 30 kpc, but this is physically implausible.) The cumulative mass distribution for this case is shown in Figure 5 as “minimum” halo. Using this curve, we find that the *minimum amount of dark matter associated with NGC 3198 inside 50 kpc is probably at least 6 times larger than the amount of visible matter*; thus, for the galaxy as a whole $M/L_B \geq 25 M_{\odot}/L_{B_{\odot}}$.

We now consider a family of disks with $V_{\text{max}} < 140 \text{ km s}^{-1}$

(all with the same scale length); the previous results apply to the maximum disk case. We find that for each of the disks with $V_{\text{max}} < 140 \text{ km s}^{-1}$ it is possible to find a matching halo described by equation (1), such that the sum of disk and halo fit the observed rotation curve. As an example, Figure 7 shows a disk with mass equal to 30% of the maximum disk allowed. Even a fit with a halo only cannot be rejected straight away (see Fig. 8). The main difference between the resulting halos is the core radius: it decreases from $R_{\text{core}} = 12$ kpc for the maximum disk mass case to 1.7 kpc when the dark halo dominates. The exponent γ of the halo density distribution is always close to 2.

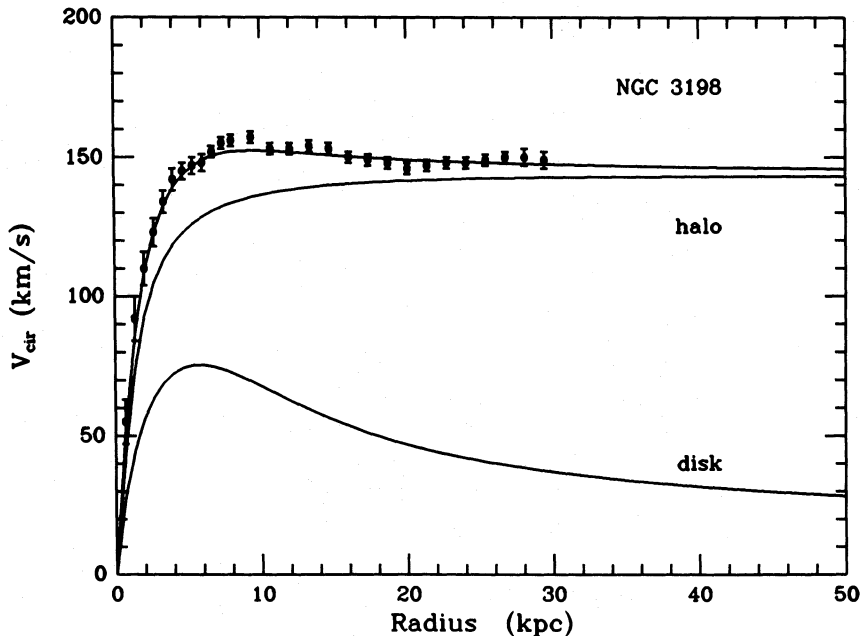


FIG. 7.—Fit of exponential disk with $M = 0.3 \times M_{\text{disk}}^{\text{max}}$ and halo to observed rotation curve. The parameters for the halo are $a = 1.3$ kpc, $\gamma = 2.05$, $\rho(R_0) = 0.0063 M_{\odot} \text{pc}^{-3}$.

c) *An Upper limit for the Mass of the Spheroidal Component*

Bulges of late-type spirals have fairly small effective radii (Simien and de Vaucouleurs 1983). Since there is also a rough relation between bulge luminosity and effective radius (Kormendy 1982), the effective radius of the bulge of NGC 3198 is expected to be quite small: $R_{\text{eff}} \approx 3$ kpc is a generous upper limit. We obtain an upper limit for the mass of a possible bulge component. The result is

$$M_{\text{sph}}(30 \text{ kpc})/M_{\text{tot}}(30 \text{ kpc}) < 0.026 ,$$

and

$$M_{\text{sph}}(30 \text{ kpc})/M_{\text{disk}}^{\text{max}}(30 \text{ kpc}) < 0.12 .$$

In reality the mass of the spheroidal component is probably much smaller, since in the optical data (Cheriguene 1975), which covers the region inside $2'$ all the way to the center, there is no indication of a more rapid rise of the rotation in the innermost region.

Although the above upper limits are not very stringent, they indicate that our conclusions in the preceding paragraph are not affected by uncertainties regarding the properties of the spheroidal component.

V. DISCUSSION

An important question left unanswered by the preceding analysis is the value of M/L for the disk. Should one seriously consider the case where the amount of visible matter is negligible with respect to the amount of dark matter (Fig. 8)? Or is the maximum disk case (Fig. 6) closer to the truth? There are three suggestive (but not definitive) reasons that support the latter possibility: (i) Measurement of mass and luminosity density in the solar neighborhood yields $M/L_V = 3.1 \pm 0.6 M_{\odot}/L_{V_{\odot}}$ (Bahcall 1984). This value of M/L includes the dark material that must reside in the disk. The uncertainty represents an effective 95% confidence level. For NGC 3198 $M/L_V(\text{disk}) \leq 4.4 M_{\odot}/L_{V_{\odot}}$. (Note that M/L is proportional to the Hubble constant.) (ii) The shape of the rising part of the

rotation curve agrees with that expected for a disk with scale length as given by the distribution of light. If the rotation curve were determined by the dark halo, such agreement would be a coincidence. (iii) The close relationship between luminosity of spiral galaxies and maximum circular velocity, implied by the small scatter in the Tully-Fisher relation, indicates that it is, after all, the amount of *visible* matter that determines the maximum rotation velocity in a galaxy. If this were not the case the amount of dark matter inside, say, 2.5 disk scale lengths must be related in a unique way to the amount of visible matter. (This may not be a strong argument; see Appendix.)

Of course, the overall flatness of rotation curves *also* implies a relation between the distributions of dark and visible matter. Indeed, indications at present are that rotation curves for spiral galaxies of all types *and* luminosities are approximately flat, or slightly rising, beyond the turnover radius of the disk (see Carignan 1983 and Carignan and Freeman, in preparation, for late-type spirals with M_B in the range -16 to -18). It is not clear yet whether this implies that the distributions of dark and visible matter are *closely* related. In analogy with the disk-bulge case, for which it is relatively easy to produce a flat rotation curve by combining a declining curve for the bulge with a rising one for the disk, it is also not difficult to make an approximately flat rotation curve beyond the turnover point of the disk by combining the declining curve for the disk with a rising one for the dark halo (see BSS). Carried one step further, this reasoning might be used as an argument against the motivation for Milgrom's (1983) proposal that Newtonian dynamics must be modified. Given the existence of dark halos, "fine tuning" of their properties to those of the visible matter would only be required if rotation curves of many galaxies turn out to be strictly flat until far beyond the turnover radius of the disk, like in NGC 3198.

So far we have assumed that dark halos are spherical. Alternatively, one might conjecture that the mass-to-light ratio of visible matter in disks of spiral galaxies increases with radius.

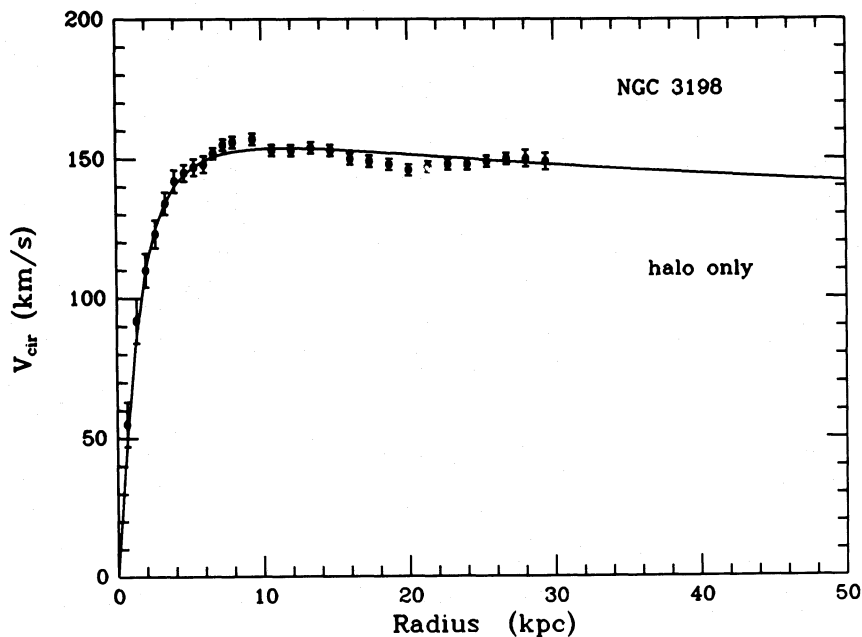


Fig. 8.—Fit of halo without disk; $a = 1.5$ kpc, $\gamma = 2.25$, $\rho(R_0) = 0.0074 M_{\odot} \text{pc}^{-3}$.

In this case our calculations regarding the amount of dark matter require a downward adjustment by a factor of ~ 1.5 . It is too early to rule out this possibility, but a comparison of the thickness of H I layers with the vertical velocity dispersion of H I in face-on galaxies suggests that the dark matter needed to make the rotation curve flat cannot be hidden in a disk (van der Kruit 1981; Casertano 1983).

After these general remarks let us return to the case of NGC 3198. Having established that there is a large amount of dark matter, one would like to know its spatial distribution. Unfortunately, the present data provide only weak constraints. If we assume the mass of the disk to lie between 0.6 and 1 times the maximum value allowed by the rotation curve, we find that the core radius of the halo lies in the range 1.8–12.5 kpc. It is also of interest to compare the volume density of dark matter in the spherical halo with the density of H I. The surface density of H I outside $R = 6'$ can be represented by

$$\mu_{\text{HI}}(R) \approx 120e^{-R/h_{\text{HI}}} M_{\odot} \text{ pc}^{-2},$$

where $h_{\text{HI}} = 3'.3$. Consider a point in the midplane of H I at a distance of 27 kpc from the center, i.e., close to the outermost point on the rotation curve. Assume that the vertical density distribution of H I at this location is Gaussian with a 1σ scale height of 1 kpc. For the halo density law used for the maximum

disk case (Fig. 4) we then find that the H I density *in the plane* still exceeds the density of dark matter by a factor of 8!

Finally, one may ask: how unique is NGC 3198? Can the results be generalized? The only unique feature that we know about for this galaxy is its relatively undisturbed, extended H I envelope, which allows an accurate determination of the rotation curve to large galactocentric distances. In other respects NGC 3198 is normal; in particular, its mass-to-light ratio inside one Holmberg radius is typical of Sc galaxies (Faber and Gallagher 1979). We thus have a solid determination of $M/L_B = 18 M_{\odot}/L_{B\odot}$ inside 30 kpc, and a speculative estimate $M/L_B \geq 25$ inside 50 kpc, that are probably typical for galaxies of this morphology. The high M/L values found for binary galaxies (see the discussion by Faber and Gallagher) are therefore consistent with those for single galaxies over the same length scale.

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APPENDIX

DARK MATTER AND THE TULLY-FISHER RELATION

Below we discuss, in a semiquantitative way, the effect of dark matter on the relation between luminosity and 21 cm profile width (T-F relation; Tully and Fisher 1977). We use the T-F relation in the following form:

$$L \propto V^{\alpha}, \quad (\text{A1})$$

where L is the total luminosity in a given wave band and V is the maximum circular velocity in the galaxy outside the bulge. Maximum circular velocity and profile width are, apart from a scale factor, nearly identical, as long as H I is present in the region around 2–3 disk scale lengths from the center. The value of α depends on the wave band and on the sample choice. For Sb–Sc galaxies, $\alpha \approx 3.5$ in B and $\alpha \approx 1.3$ in H (Aaronson and Mould 1983).

On the assumption that a galaxy consists of three components, a spheroid, an exponential disk, and a dark halo, equation (A1) can be rewritten as follows:

$$L \propto \{G[0.39M_{\text{disk}} + 0.45M_{\text{sph}}(2.2h) + 0.45M_{\text{halo}}(2.2h)]/h\}^{\alpha/2}, \quad (\text{A2})$$

where h is the disk scale length. Here we have made the additional assumptions that both spheroid and dark halo are spherical and that the location of maximum circular velocity coincides with the turnover point of the rotation curve of the disk. For reasonable choices of disk scale length and effective radius of the spheroid, most of the spheroid mass lies inside $2.2h$. Thus, for disk-dominated systems, we make a very small error—on the percent level—if we replace $0.45M_{\text{sph}}(2.2h)$ in equation (A2) by $0.39M_{\text{sph}}(\infty)$. [In fact, for 15 galaxies in Boroson 1981 with known h and R_{eff} , the median value of $0.45M_{\text{sph}}(2.2h)$ is $0.42M_{\text{sph}}(\infty)$.] Then

$$L \propto \{[M + 1.2M_{\text{halo}}(2.2h)]/h\}^{\alpha/2}; \quad (\text{A3})$$

M is the total visible mass of the galaxy, equal to $M_{\text{disk}} + M_{\text{sph}}$.

If the amount of dark matter inside 2.2 disk scale lengths is negligible, the T-F relation takes the simple form:

$$L \propto (M/h)^{\alpha/2}. \quad (\text{A4})$$

Such a relation is easy to understand if mass-to-light ratio and disk scale length vary with mass (see Burstein 1982). Adopting power-law relations, $M/L \propto M^a$ and $h \propto M^b$, one finds

$$\alpha = 2(1 - a)/(1 - b). \quad (\text{A5})$$

For a family of galaxies with disks with constant central surface density, $b \approx 0.5$ ($b = 0.5$ if there is no spheroidal component). Then $\alpha \approx 4(1 - a)$, and for M/L independent of mass, $\alpha \approx 4$. This is the original explanation of Aaronson, Huchra, and Mould (1979) for the exponent 4 of the Tully-Fisher relation in the near-infrared.

Equation (A3) shows that the small observed spread in the T-F relation implies either a negligible amount of dark matter inside 2.2 disk scale lengths, or a correlation between the amounts of dark and visible matter. Such a correlation is indeed built-in: since

disk scale length is known to depend on mass, the amount of dark matter inside 2.2 disk scale lengths also depends on mass. Thus, a relation between mass, luminosity, and disk scale length among galaxies as given in equation (A4) may not be destroyed by the presence of a moderate amount of dark matter, even if the density distribution of dark matter is largely independent of the visible matter.

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