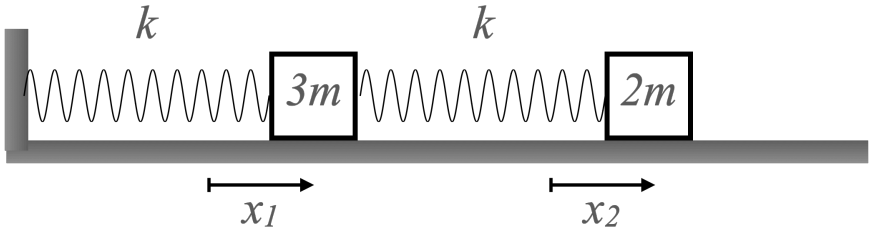
Name: _____

PHYS2502 Mathematical Physics Quiz #10 7 Apr 2022

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

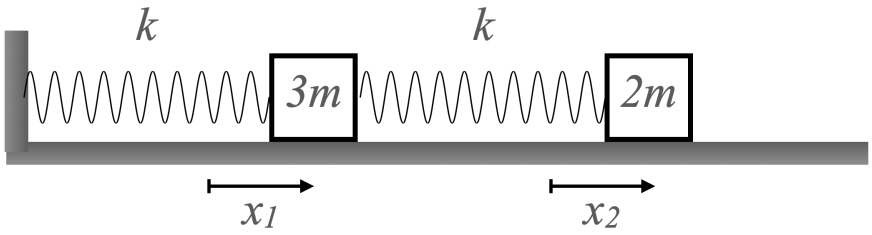
Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Find the eigenfrequencies of the coupled oscillator system below



PHY2502 Mathematical Physics Quiz #10 Solution

Find the eigenfrequencies of the coupled oscillator system below



You did this problem the hard way in HW#6 Problem#5. The equations of motion are

$$\begin{aligned} 3m\ddot{x}_1 &= -kx_1 + k(x_2 - x_1) = -2kx_1 + kx_2 \\ 2m\ddot{x}_2 &= -k(x_2 - x_1) = kx_1 - kx_2 \end{aligned}$$

Inserting the standard ansatz, and writing using vectors and matrices, this becomes

$$\underline{\underline{\Omega}}\,\underline{x} = \frac{\omega^2}{\omega_0^2}\,\underline{x} \qquad \text{where} \qquad \underline{\underline{\Omega}} = \begin{bmatrix} 2/3 & -1/3 \\ -1/2 & 1/2 \end{bmatrix}$$

The characteristic equation for the eigenvalues $\lambda = \omega^2/\omega_0^2$ is

$$\left(\frac{2}{3} - \lambda\right)\left(\frac{1}{2} - \lambda\right) - \frac{1}{6} = \frac{1}{6}\left[(2 - 3\lambda)(1 - 2\lambda) - 1\right] = \frac{1}{6}\left[1 - 7\lambda + 6\lambda^2\right] = \frac{1}{6}\left[(1 - \lambda)(1 - 6\lambda)\right] = 0$$

so the eigenvalues are $\lambda = 1$ and $\lambda = 1/6$. The eigenfrequencies are

$$\omega^{(\lambda=1)} = \omega_0 \qquad \text{and} \qquad \omega^{(\lambda=1/6)} = \frac{1}{\sqrt{6}}\omega_0$$