Name: _____

PHYS2502 Mathematical Physics Quiz #8 24 Mar 2022

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

This quiz has two parts, each worth 10 points.

(1) Find the solution $u(x, y)$ for the partial differential equation and boundary condition

$$\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y} \qquad u(x, 0) = 2x$$

(2) Find the Fourier Transform $A(k)$ of the function $f(x) = ae^{-\beta|x|}$ were $\beta > 0$. You are welcome to make use of the definite integral

$$\int_0^\infty e^{-\beta x} \cos(kx)\, dx = \frac{\beta}{\beta^2 + k^2}$$

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(1) By inspection, the general solution is $u(x, y) = f(x - y)$ for some function $f(z)$. The boundary condition gives $u(x, 0) = f(x) = 2x$. Therefore, the solution is $u(x, y) = 2(x - y)$.

(2) Just apply the definition of the Fourier Transform and make use of the fact that $f(x)$ is an even function, as is $\cos(kx)$, whereas $\sin(kx)$ is odd. You have

$$\begin{aligned} A(k) &= \int_{-\infty}^{\infty} e^{-ikx} f(x) = \int_{-\infty}^{\infty} [\cos(kx) - i \sin(kx)]\, ae^{-\beta|x|}\, dx \\ &= 2a \int_0^\infty \cos(kx) e^{-\beta x}\, dx = \frac{2a\beta}{\beta^2 + k^2} \end{aligned}$$