Name: _____

PHYS2502 Mathematical Physics Quiz #7 8 Mar 2022

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

(1) Calculate the gradient of the scalar field $f(x, y, z) = xyz$.

(2) Calculate the divergence of the vector field $\vec{v}(x, y, z) = \hat{i}x + \hat{j}z - \hat{k}y$.

(3) Calculate the curl of the vector field $\vec{v}(x, y, z) = \hat{i}x + \hat{j}z - \hat{k}y$.

PHY2502 Mathematical Physics Quiz #7 Solution

(1) Calculate the gradient of the scalar field $f(x, y, z) = xyz$.

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(1) Gradient.

$$\vec{\nabla} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z} = \hat{i}yz + \hat{j}xz + \hat{k}xy$$

(2) Divergence.

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 1 + 0 + 0 = 1$$

(3) Curl.

$$\begin{aligned} \vec{\nabla} \times \vec{v} &= \hat{i}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{j}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{k}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \\ &= \hat{i}(-1-1) + \hat{j}(0-0) + \hat{k}(0-0) = -2\hat{i} \end{aligned}$$