Name: ______________________________

PHYS2502 Mathematical Physics Quiz #1 13 Jan 2022

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

The so-called "Planck Mass" is the mass (or energy, when multiplied by c^2) at which gravity is unified with quantum mechanics and special relativity. Therefore, it must depend on (the reduced) Planck's constant $\hbar$, the speed of light c, and Newton's gravitational constant G. Use dimensional analysis to arrive at an expression for the Planck Mass M_P. It will help to know that $\hbar$ has units of angular momentum, and that the force of gravity between two masses m_1 and m_2 separated by a distance r is Gm_1m_2/r^2.

PHY2502 Mathematical Physics Quiz #1 Solution

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$$[\hbar] = L \cdot MLT^{-1} = L^2MT^{-1}$$
$$[\text{force}] = MLT^{-2} = [G]M^2L^{-2} \qquad \text{so} \qquad [G] = L^3M^{-1}T^{-2}$$

Writing $M_P = G^x\hbar^y c^z$ means that

$$M = L^{3x}M^{-x}T^{-2x}L^{2y}M^yT^{-y}L^zT^{-z} = L^{3x+2y+z}M^{-x+y}T^{-2x-y-z}$$

The mass factor says that $y = x + 1$, so the length and time factors together say that

$$\begin{aligned} 3x + 2y + z = 5x + 2 + z = 0 \qquad &\text{so} \qquad 5x + z &= -2 \\ -2x - y - z = -3x - 1 - z = 0 \qquad &\text{so} \qquad 3x + z &= -1 \end{aligned}$$

Subtract these to get $2x = -1$ or $x = -1/2$. Then $y = x + 1 = 1/2$. There are several equations that can give you z, for example $z = --1 - 3x = -1 + 3/2 = 1/2$. Therefore

$$M_P = G^{-1/2}\hbar^{1/2}c^{1/2} = \left(\frac{\hbar c}{G}\right)^{1/2}$$

which is verified easily enough with a quick Google search.