PHYS2502 Mathematical Physics Lab #11 4 Apr 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

This exercise is to find the principle axes and moments of inertia for a "rigid body" that consists of eight equal masses m at the corners of a rectangular block in three dimensional space. You will also get some experience here in making plots in three dimensions.

The corners are at $(x, y, z) = (0, 2a, \pm b)$, $(-a\sqrt{3}, a, \pm b)$, $(a\sqrt{3}, -a, \pm b)$, and $(0, -2a, \pm b)$. Write a notebook that follow and documents these steps.

(1) Define some named variable that is a list of the vectors for each of the eight corners.

(2) Pick some values for a and b and plot the eight corners in 3D. Draw lines along the x, y, and z axes, and label them. You can combine the different graphics objects with Show. You'll see that you can use the cursor to move around the direction from which you view your picture. Convinced yourself you did this right, and the eight points form the corners of a rectangular block.

Use Graphics3D[{Red, PointSize[0.05], Point[v]}], or something like it, to plot the corners, where v is your named variable of the vectors. You can play with other shapes instead of Point, if you want. I suggest you include the options Axes→True and ViewProjection→"Orthographic" to make the image easier to understand. You can draw the axes using ParametricPlot3D, and Graphics3D[Text["X", {xval, 0, 0}]] will put the text "X" somewhere on the x-axis.

(3) The center of mass is located at the vector $\vec{r}_{\rm cm} = \sum_\alpha m_\alpha \vec{r}_\alpha / \sum m_\alpha$ where α runs over the masses and, in this case, the m_i are all equal to m for the eight masses. Show that the center of mass is located at the origin for this set of positions. You will find Total useful for calculating the sum.

(4) Compute the matrix that represents the moment of inertia tensor $\underline{\underline{\mathcal{I}}}$, and display it in standard matrix form to prove that it is symmetric. The moment of inertia is given by

$$\underline{\underline{\mathcal{I}}} = \sum_\alpha \left[m_\alpha r_\alpha^2 \, \underline{\underline{I}} - m_\alpha \, \underline{r}_\alpha \, \underline{r}_\alpha \right]$$

where $\underline{\underline{I}}$ is the identity matrix, r_α^2 is the square of the distance from the origin to mass α, and $\underline{r}_\alpha \, \underline{r}_\alpha$ is the dyadic formed from the position vector $\underline{r}_\alpha$ of mass α. Something like

Simplify[Table[Norm[v[[i]]], {i, 1, 8}]]

will give you a list of the eight values of r_α^2. Something similar using TensorProduct will construct the dyadic from the position vectors.

(5) Find the eigenvalues of the inertia tensor. These are the principal moments of inertia.

(6) Find the principal axes for this rigid body, given by the eigenvectors of the inertia tensor. Add lines in the directions of the principal axes to your plot from (2). Use a different color for the principal axes so that you can see them clearly, and convince yourself that these are indeed the "symmetry" axes of this collection of masses.