PHYS2502 Mathematical Physics Lab #10 28 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed Mathematica *notebook to the Canvas page for the course.*

This is a simple lab, just aiming to get you some experience in manipulating vectors and matrices in Mathematica. These are not fundamental entities in Mathematica, but are just "lists", that is, enclosed in curly brackets. What matters are the functions you invoke on them, which can interpret these special lists as vectors or matrices.

You'll be working with the following:

$$\underline{u} = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 2 \end{bmatrix} \quad \underline{v} = \begin{bmatrix} i \\ 4 \\ 1-i \\ -1 \end{bmatrix} \quad \underline{\underline{A}} = \begin{bmatrix} 1 & 5 & 7 & -3 \\ -3 & 8 & 4 & 2 \\ -7 & 5 & 7 & 0 \\ 3 & 6 & 5 & 4 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 0 & 9 & -3-i & 9 \\ 9 & 6 & 2+i & 4 \\ -3+i & 2-i & 12 & -7-i \\ 9 & 4 & -7+i & -4 \end{bmatrix}$$

(1) Enter the two vectors and two matrices above as lists, and produce their output in the standard format using MatrixForm.

(2) Form the dual vector $\underline{\tilde{v}}$ and give its output in the standard format. For this, you will need the function ConjugateTranspose.

(3) Prove that $\underline{\underline{B}}$ is Hermitian using the function HermitianMatrixQ.

(4) Find the square of the norm of $\underline{v}$, that is calculate $\underline{\tilde{v}v}$, and confirm that it is positive definite. You can use Dot, or the shorthand ".", to calculate the sum of the one-by-one product of the elements.

(5) Calculate the product $\underline{\underline{A}}\,\underline{u}$ and give the output in standard format.

(6) Calculate the product $\underline{\tilde{v}}\,\underline{\underline{B}}$ and give the output in a simple horizontal list format. (If you see a nice way to give the output in the form of a standard row vector, let me know!)

(7) Show that $\underline{\underline{A}}$ and $\underline{\underline{B}}$ do not commute. It is easiest to do this using the logical "==" operation which just returns "True" or "False".

(8) Find the inverse $\underline{\underline{A}}^{-1}$ of the matrix $\underline{\underline{A}}$ and output in the standard format. (Warning: The inverse matrix is ugly in this example.) Show that $\underline{\underline{A}}^{-1}\underline{\underline{A}}$ and $\underline{\underline{A}}\,\underline{\underline{A}}^{-1}$ give the right answer.