PHYS2502 Mathematical Physics Lab #8 14 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

This lab concerns a numerical solution to a partial differential equation, namely the transverse vibrational motion of a stretched string, fixed at both ends, as a function of time. In the end, you will make an animation of the motion.

Transverse vibrations of a stretched string follow from the wave equation, namely

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ is the shape of the string for a longitudinal position x at time t. Consider a string that has the shape $u(x,0) = (L/2 - x)^2(L/2 + x)^2$ at $t = 0$, where the string extends from $x = -L/2$ to $x = L/2$ and is fixed at the endpoints, that is $u(\pm L/2, t) = 0$. Also assume the string is not moving at $t = 0$.

Use NDSolve to find $u(x,t)$. For the numerical calculation, just set L and v to some numerical values. You will of course need to include the initial and boundary conditions that the string is fixed at both ends, starts from rest, and has the prescribed initial shape. Use something like "Derivative[0, 1][u][x, 0] == 0" to specify the initial condition that the string does not move. The initial shape I specified here has no sharp changes, and this is useful for achieving a numerical solution to the differential equation with default options.)

You'll need to specify some maximum time over which to do the calculation. Pick a reasonable value, given that the relevant time scale is L/v.

You may get some warnings about numerical accuracy. You probably don't need to worry about these, but you are welcome to modify the default options in NDSolve to handle them.

Check your result by plotting $u(x,t)$ for different times t, and see that it moves the way you expect. I would also like you to put the plot inside Animation to watch the string move in real time. Check the MATHEMATICA documentation for options on how to use Animation.