PHYS2502 Mathematical Physics Lab #7 7 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

This lab consists of a few simple exercises with scalar and vector fields and their derivatives.

(1) Start with the scalar field $f(x,y) = (x^2 + y^2)/2$.

- Use Grad to find $\vec{v} = \vec{\nabla} f$ and confirm it is what you expect.
- Use Show to combine a ContourPlot of $f(x,y)$ with a VectorPlot of $\vec{v}(x,y)$. It will be clearer if you turn off the ContourShading. You should also include the PlotLegends for the vector plot.
- Find $\vec{\nabla} \cdot \vec{v}$ using Div and confirm the result is what you expect.
- Calculate the Curl of $\vec{v}$ and confirm the result is what you expect. For this, you need a three-dimensional version of $\vec{v}$ that has z-component equal to zero. The command Flatten[{v, 0}] will do this for you, but you can also just do it by hand.

(2) Now use the scalar field $f(x,y,z) = -1/\sqrt{x^2 + y^2 + z^2}$.

- Calculate $\vec{v} = \vec{\nabla} f$ and again make a combination contour and vector plot. You will need to make two-dimensional versions of the fields for this. Do this by setting replacing z with 0 in the expressions and only take the first two parts of $\vec{v}$. See the documentation for Part if you don't know how to do this. When you make the plots, it will be best to avoid $(x,y) = (0,0$, so just plot in the upper right quadrant.
- Calculate $\vec{\nabla} \cdot \vec{v} = \vec{\nabla}^2 f$. Are you surprised at the answer?
- What is the more or less obvious physical situation described by these fields? Consider using the function ToPolarCoordinates of the two-dimensional vector field.

(3) Finally, start with the vector field $\vec{v}(x,y) = (-\hat{i}y + \hat{j}x)/(x^2 + y^2)$.

- Make a vector plot of this field
- Calculate the curl of this field and simplify the result. Are you surprised at the answer?
- What is the more or less obvious physical situation described by these fields? Consider using the function ToPolarCoordinates on the field.