PHYS2502 Mathematical Physics   Lab #6   21 Feb 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

In class, and in the notes, we solved the coupled differential equations problem for two (equal) masses and three (equal) springs using the harmonic ansatz and algebra. This is of course not always possible for coupled differential equations, so we have to resort to other techniques. This lab is about using MATHEMATICA to solve the same equations we solved in class, but solve them directly using the DSolve function. You will also apply this approach to the equations from Problem 5 on Homework 6, so you can use this to check your homework solution.

Equations (3.31) in the notes (as I write this) are the differential equations that govern two identical masses connected by three identical springs, shown in Figure 3.11. Find the solutions $x_1(t)$ and $x_2(t)$ directly using DSolve. You need to provide as input the two differential equations, plus the four initial conditions, in a list as the first argument of DSolve. Use arbitrary values for the initial conditions.

Figure 3.12 is made with initial conditions $x_1(0) = 1$ and $\dot{x}_1(0) = 0 = x_2(0) = \dot{x}_2(0)$. Reproduce both plots in this figure, that is $x_1(t)$ and $x_2(t)$ in one plot, and $x_1(t) \pm x_2(t)$ in the other. For the second plot, you might want to use ListLinePlot combined with your Plot function, using the Show function, to make dashed vertical lines indicating what you expect to be the two periods corresponding to the eigenfrequencies to compare with your plots.

Now solve the differential equations you derived for Problem 5 on Homework 6, and once again plot $x_1(t)$ and $x_2(t)$. Also plot the linear combinations you figured out for the eigenfrequencies, and again show that the periods agree with what you calculated algebraically.