PHYS2502 Mathematical Physics   Lab #5   14 Feb 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

Happy Valentine's Day!

Consider the nonlinear ordinary second order differential equation for the function $x(t)$

$$\frac{d^2x}{dt^2} = x^2 - x$$

Physically, you can think of this as Newton's Second Law for the position $x(t)$ of some particle moving in one dimension, where position and time are scaled to be dimensionless. In this case, the "force" $F(x) = x^2 - x$.

Try solving this equation using DSolve, with the initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$. You won't succeed, but see what happens. This equation has no analytic solution, and must be solved numerically.

The MATHEMATICA function for numerical solutions to differential equations is NDSolve. The syntax is the same as DSolve except that when you specify the independent variable, t in this case, you have to provide a list with the minimum and maximum values.

(1) Find the numerical solution and plot the result for $0 \leq t \leq 10$, using the initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$. The result might confuse you. If so, move on to (2).

(2) Repeat for initial conditions $x(0) = 0.1$ and $\dot{x}(0) = 0$. This plot should look less confusing. Try also $x(0) = -0.1$ and plot the two on the same graph. It should start becoming clear to you what is going on.

(3) Repeat for initial conditions $x(0) = \pm 0.5$ and $\dot{x}(0) = 0$. You might want to increase the range for solving and plotting to $0 \leq t \leq 25$ to avoid some confusion.

(4) Finally, use initial conditions $x(0) = -0.51$ and $\dot{x}(0) = 0$. Note that you can change the vertical range of the plot using PlotRange→{min,max} for appropriate minimum and maximum values. You might also want to play around with the upper end of the time range.

Explain what is going on here, physically. What is so special about the point $x = -0.5$? You will find it useful to plot the potential energy function $V(x) = -\int F(x)dx$.

This problem illustrates an example of something called the *nonlinear oscillator*.