PHYS2502 Mathematical Physics Lab #4 7 Feb 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

In class we solved the differential equation and initial conditions that describe vertical speed of a mass that falls from rest and is subject to a drag force linear in its velocity. This was a first order differential equation.

This lab is to directly solve, using MATHEMATICA, the differential equation for the vertical position, a second order differential equation, subject to arbitrary initial conditions. Then you are to check your answer against what you expect if there is no drag, and also to check that your solution agrees with the velocity we found in class.

Set up your differential equation for the vertical position $y(t)$ of a mass m subject to a drag force $b\dot{y}(t)$, and solve it using DSolve, with the initial conditions $y(0) = y_0$ and $\dot{y}(0) = v_0$. Look at the documentation to understand the syntax. There are different ways to set this up, but I am partial to

sol = DSolve[DiffEq, y[0] == y0, y'[0] == v0, y[t], t];

where DiffEq is the differential equation for vertical position $y(t)$. Used in this form, sol is a replacement statement for the function y[t]. Writing y = y[t] /. sol[[1]] gives you the height y in a useful format. Then do the following:

(1) Confirm that your result for y reduces to the correct result when $b = 0$. You won't be able to just set $b = 0$, so figure out a different way to carry this out. There are a few options.

(2) Use your solution to find the velocity, and confirm that you get the same result as we did in class for $v_0 = 0$.

(3) Find the time it takes for the mass to go up from $y_0 = 0$ and then come back down to where it started. I think you'll find the solution in terms of something called ProductLog which you might care to look up. Then expand this result for small values of b. Is the first term in the expansion correct, that is, the term that does not contain b?

I remind you that it is always a good idea to define the assumptions inherent in your equations, as this usually avoids problems with integrations. For this lab, a statement like

$Assumptions = {m>0, g>0, b>0, y0>0, v0>0};

would be useful.