PHYS2502 Mathematical Physics Lab #3 31 Jan 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your executed MATHEMATICA *notebook to the Canvas page for the course.*

The total mechanical energy of a simple pendulum of length ℓ and bob mass m is

$$E = \frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell(1-\cos\theta) = mg\ell(1-\cos\theta_0)$$

where $-\theta_0 \le \theta \le \theta_0$ is the angle through which the pendulum swings, and $\dot{\theta} \equiv d\theta/dt$. Find an integral for the period $T(\theta_0)$, divided by the "small angle" period that you learned in your first Physics course. This is most easily done by integrating dt from $\theta = 0$ to $\theta = \theta_0$ and then multiplying by four.

(1) Let MATHEMATICA evaluate the integral for you. You'll get a peculiar-looking result in terms of an "elliptic integral." Plot the result as a function of θ_0, from zero up to something very close to π. You should see something close to unity for "small" θ, but then increasing for large θ. (What is the period if you reach $\theta_0 = \pi$?)

(2) You can also do the integral numerically, with the function `NIntegrate`, for any specific value of θ_0. Check that the numerical integration and the expression in terms of the elliptic integral agree for various values of θ_0. You might want to define a list of θ_0 values, and then use `Table` to tabulate and compare the two approaches. The function `Grid` is a handy way to print out the values of the table.

(3) By making the substitution $\sin(\theta/2) = Au$ where $A = \sin(\theta_0/2)$, your integral for the scaled period becomes

$$T(\theta_0) = \frac{2}{\pi}\int_0^1 \frac{1}{\sqrt{1-u^2}}\frac{1}{\sqrt{1-A^2u^2}}\,du$$

(You're not required to derive this for this exercise, but it would be good practice for you to go through it anyway, either inside or outside MATHEMATICA.) Integrating this will give you the result in terms of a different elliptic integral. Compare some values to your earlier result to confirm that this expression is correct. Then use the `Series` function to expand $T(\theta_0)$ to lowest nonzero order in $A = \sin(\theta_0/2)$. Numerically compare the truncated expansion to the exact formula for values up to $\theta_0 = 1$. How good is this approximation? You might want to plot the exact solution against the truncated expansion over the range $0 \le \theta_0 \le \pi$.