PHYS2502 Mathematical Physics Homework #13 Due 19 Apr 2022

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) This problem involves functions that are not analytic everywhere.

(a) A complex function $f(z) = 2y + ix$ where $z = x + iy$. Use the definition of the derivative directly to show that $f'(z)$ does not exist anywhere in the complex plane. Then show that this is consistent with the Cauchy-Riemann relations.This problem involves functions that are analytic.

(b) A complex function $f(z) = |x| - i|y|$ where $z = x + iy$. Where, if anywhere, in the complex plane is this function analytic?

(2) This problem involves functions that are analytic.

(a) Prove that $f(z) = e^z$ is analytic everywhere in the complex plane.

(b) Show that if $f(z)$ is an analytic function of z, then $g(z) = zf(z)$ is also an analytic function of z. Use this to explain why

$$f(z) = \sum_{n=0}^{\infty} c_n z^n$$

is an analytic function of z.

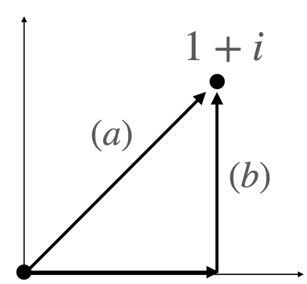
(3) Find an analytic function $f(z) = u(x, y) + iv(x, y)$ whose imaginary part is

$$v(x, y) = (y \cos y + x \sin y)e^x$$

(4) Referring to the diagram on the right, calculate the integral

$$\int_0^{1+i} (z^2 - z)\, dz$$

along the paths (a) and (b), where (b) is a horizontal step followed by a vertical step. Explain why the two results compare to each other the way that they do.



(5) Calculate the integral

$$\int_0^{\infty} \frac{1}{x^2 + 1} dx$$

in two different ways. First, use the substitution $x = \tan\theta$, and second as a contour integral in the complex plane. You can use MATHEMATICA to check your answer.