PHYS2502 Mathematical Physics Homework #12 Due 12 Apr 2022

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) A particle of mass m moves in one dimension x over some time interval $t_1 \leq t \leq t_2$, under the influence of a force $F(x) = -dV/dx$, some function $V(x)$. Show that finding the function $x(t)$ which minimizes

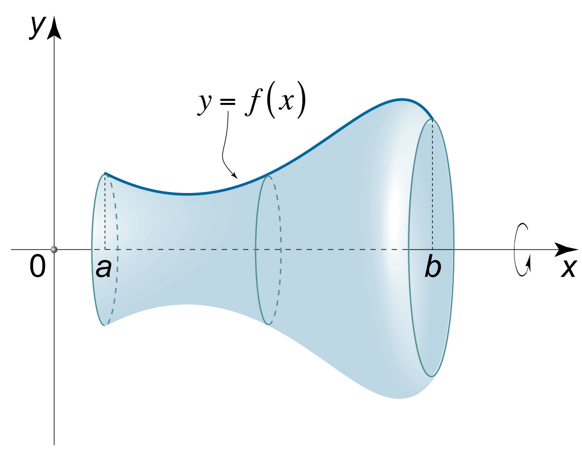
$$S = \int_{t_1}^{t_2} L(x, \dot{x})\, dt \qquad \text{where} \qquad L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - V(x)$$

is the same as showing that $x(t)$ is determined by Newton's Second Law of Motion. Then show that since L does not explicitly depend on t, total mechanical energy is conserved.

(2) We discussed in class two special cases which allowed the Euler-Lagrange equations to be integrated once. In fact, when we first started talking about the Calculus of Variations, we found that a straight line gave the shortest distance between two points, using the first of these special cases. Show that the functional for the shortest path between two points is also an example of the second special case, and use that form to show that solution is a straight line. (As I write this, the second case is shown in Equation 7.7, Section 7.2.1 in the course notes.)

(3) Use Mathematica to find and plot the brachistochrone solution for a bead starting at the origin and ending at $(a, b) = (1, 2)$.

(4) A "surface of revolution" is formed when a shape given by $y = f(x)$ for $a \leq x \leq b$ is rotated about the x-axis, as shown in the figure. Find the form of thefunction $f(x)$ which minimizes the surface area. You don't need to solve for the constants of integration in terms of the fixed points of $f(x)$ at $x = a$ and $x = b$.



(5) A chain of length $L > 2a$ hangs freely between two points $x = \pm a$ on the x-axis in the xy plane. Find the equation $f(x)$ that describes the resting shape of the chain, assuming that this is the shape that minimizes the center of gravity. Of course, the length L must remain fixed, and $f(\pm a) = 0$. Eliminate whichever constants of integration are easiest, but come up with a simple physical interpretation of the Lagrange multiplier used to set the length constraint.