PHYS2502 Mathematical Physics Homework #11 Due 5 Apr 2022

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Use the properties of determinants to prove that $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{0}}$ implies that either $|\underline{\underline{A}}| = 0$ or $|\underline{\underline{B}}| = 0$, or both, where $\underline{\underline{0}}$ is the matrix of all zeros. Demonstrate this with the matrices

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad \text{and} \qquad \underline{\underline{B}} = \begin{bmatrix} a & b \\ -a & -b \end{bmatrix}$$

where a and b can be any complex numbers. Which matrix has zero determinant?

(2) Follow the procedure we went through in class to find the symmetry axes of the conic section $6x^2 + 12xy + y^2 = 16$, and find the angle they make with the x, y axes. What kind of curve is this? A plot would be helpful. You can do this with MATHEMATICA if you want, but the necessary algebra is rather simple.

(3) Find the eigenvalues and eigenvectors for the matrix

$$\underline{\underline{\sigma}}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

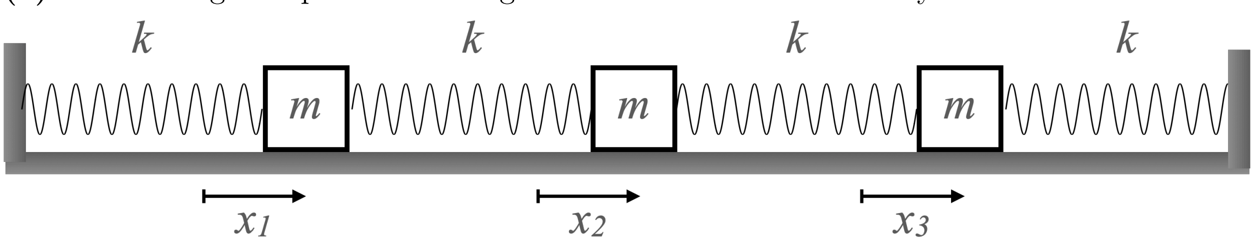
one of the three *Pauli matrices*. Do this by hand, not with MATHEMATICA. Normalize the eigenvectors and show that they are orthogonal.

(4) Find the eigenvalues, two of which equal each other, of the real symmetric matrix

$$\underline{\underline{A}} = \begin{bmatrix} 13 & 4 & -2 \\ 4 & 13 & -2 \\ -2 & -2 & 10 \end{bmatrix}$$

Construct the three eigenvectors by hand, not with MATHEMATICA. You will find you have more freedom than you would have thought. Do you see how to use this freedom to make all three eigenvectors mutually orthogonal?

(5) Find the eigenfrequencies and eigenmodes for the mechanical system



Make a plot that shows the motions of each of the three masses, for the three sets of initial conditions where the masses start at rest with position given by each of the three eigenvectors. Briefly describe the motions of the three masses, for each of the eigenmodes.