# PHYS2502 Mathematical Physics Homework #10 Due 29 Mar 2022

*This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.*

**(1)** Find the inverse $\underline{\underline{A}}^{-1}$ for the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

by solving the system of equations $\underline{\underline{A}}\,\underline{x} = \underline{c}$ for the vector $\underline{x}$ in terms of an arbitrary vector $\underline{c}$ and expressing your result as $\underline{x} = \underline{\underline{A}}^{-1}\underline{c}$. Try using MatrixForm[Inverse[A]] in MATHEMATICA to check your answer.

**(2)** Use the orthogonality of the Legendre Polynomials to derive an infinite series that gives $f(x) = e^x$ over the domain $-1 \leq x \leq 1$ in the form of a "Shin expansion"

$$f(x) = \sum_{\ell=0}^{\infty} A_\ell P_\ell(x)$$

Plot your result for some number of terms of the series and compare to $f(x)$. Note that it is natural to carry out this calculation using MATHEMATICA, but you might be surprised at how few terms you need to get a good approximation.

**(3)** The trace $\mathsf{tr}(\underline{\underline{A}})$ of a matrix $\underline{\underline{A}}$ is defined as the sum over the diagonal elements of $\underline{\underline{A}}$, that is $\mathsf{tr}(\underline{\underline{A}}) = A_{ii}$. Prove that $\mathsf{tr}(\underline{\underline{A}}\,\underline{\underline{B}}) = \mathsf{tr}(\underline{\underline{B}}\,\underline{\underline{A}})$ for any two matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$, regardless of whether or not they commute.

**(4)** Consider rotations in 3D space, recalling how we describe rotations in a plane.

(a) Find the $3 \times 3$ matrix $\underline{\underline{A}}$ that rotates a vector by 90° around the $z$-axis.

(b) Find the $3 \times 3$ matrix $\underline{\underline{B}}$ that rotates a vector by 90° around the $x$-axis.

(c) Show that $\underline{\underline{A}}\,\underline{\underline{B}} \neq \underline{\underline{B}}\,\underline{\underline{A}}$ by explicit matrix multiplication.

(d) For the vector $\underline{v} = \hat{\jmath}$, the unit vector in the $y$-direction, calculate $\underline{\underline{A}}\,\underline{\underline{B}}\,\underline{v}$ and $\underline{\underline{B}}\,\underline{\underline{A}}\,\underline{v}$. Sketch diagrams that demonstrate these unequal results. (Don't be too concerned about the sign of the rotation angle.)

**(5)** Find the norms of the vectors $\underline{v}$ and $\underline{u}$ below, and also show that they are orthogonal to each other. Then find some vector $\underline{w}$ with unit norm that is orthogonal to both $\underline{v}$ and $\underline{u}$.

$$\underline{v} = \begin{bmatrix} i \\ 1 \\ -1 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} 2i \\ -2 \\ 0 \end{bmatrix}$$