PHYS2502 Mathematical Physics Homework #5 Due 15 Feb 2022

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Make some sketches of the motion of $x(t)$ for the damped oscillator, similar to Figure 3.2 and 3.3 in the notes. Plot against time in units of the fundamental period of the undamped oscillator. You may carry out the calculations and make the plots using MATHEMATICA or some other application. Plot the following cases:

(a) $\beta = 0.05\omega_0$, $x_0 = 5$, $v_0 = 2\omega_0$

(b) $\beta = 1.5\omega_0$, $x_0 = 1$, $v_0 = -2\omega_0$

(c) $\beta = \omega_0$, $x_0 = 1$, $v_0 = -2\omega_0$

(2) Reproduce the plots in Figure 3.5 of the notes, that is $x(t)$ for a forced damped oscillator with $\beta = 0.05\omega_0$, $\gamma = 10$, and with initial conditions $x(0) = \dot{x}(0) = 0$. The three plots are for $\omega = 0.5\omega_0$, $\omega = \omega_0$, and $\omega = 1.5\omega_0$. Time is plotted in units of the fundamental period of the undamped oscillator. You only need to make the three plots, not necessarily on the same set of axes, but don't be afraid to try making the plot this way. If you want to be ambitious, consider using the Manipulate function in MATHEMATICA to see how the plot behaves if you let ω be adjustable on a sliding scale. (This is a nice demonstration of resonance.)

(3) In the terminology we used in class and in the notes, we saw that if $\beta > \omega_0$ ("over damping"), then the solution to the damped oscillator is the sum of two exponential functions, no matter how close β is to ω_0. However, if $\beta = \omega_0$ ("critical damping"), the solution magically turns into a single exponential dependence. Write $\omega_0^2 = \beta^2(1 - \epsilon^2)$ and show that for $\epsilon \ll 1$ the over damped solution turns into the critically damped solution.

(4) Use the series approach to find the solutions for $y'' = y(x)$ and show that the result is the same as the series expansion for $y(x) = c_1 \cosh(kx) + c_2 \sinh(kx)$. How would you define constants a_1 and a_2 in terms of c_1 and c_2 so that the solution is $y(x) = a_1 e^x + a_2 e^{-x}$?

(5) Find general solution to $y''(x) = xy(x)$, which I will call "Keen's Equation", using series approach. Show that there can be no term in the series proportional to x^2, and that the recursion relations relate every third term of the expansion. Separate the two solutions you find for Keen's Equation, and explicitly indicate the constants of integration. Write out the first ten or so nonzero terms of each of the two solutions, and plot them. (Don't go too far in $\pm x$ so that you over run the range afforded by the number of terms you calculated in the expansion!) Note the difference in behavior for $x < 0$ and $x > 0$.