PHYS2502 Mathematical Physics Homework #4 Due 8 Feb 2022

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Find the solution of the differential equation $dx/dt = x^2$ where x is position and t is time, and where $x(0) = a$ where $a > 0$. For what range of times is your solution valid? *Careful! This is a trick question.*

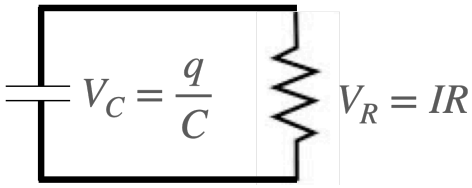
(2) The pressure $P(T)$ along a liquid-gas phase boundary on a pressure vs temperature (T) diagram is the solution to the differential equation

$$\frac{dP}{P} = k\frac{dT}{T^2}$$

where k is a a constant. If the pressure is P_0 at a temperature T_0, find the function $P(T)$.

(3) An object of mass m falls from rest some distance above the Earth's surface. It is subject to a drag force av^2 proportional to its velocity. Find its velocity $v(t)$, and check that your answer is dimensionally correct. Then check that you get the correct behavior for both short and long times. I suggest that, as we did in class, that you choose a coordinate system where "up" is positive. (If you carry out the necessary integrals using MATHEMATICA, then please also submit a PDF of your executed notebook.)

(4) The diagram at the right shows a capacitor C connected in series with a resistor R. The potential difference across the capacitor is $V_C = q/C$ where q is the charge stored on the capacitor. The potential difference across the resistor is $V_R = IR$ where $I = dq/dt$ is the current through the resistor. If the initial charge on the capacitor is q_0, find $q(t)$ as a function of time.



(5) The diagram at the right shows a capacitor C connected in series with an inductor L. The potential difference across the capacitor is $V_C = q/C$ where q is the charge stored on the capacitor. The potential difference across the inductor is $V_L = LdI/dt$ where $I = dq/dt$ is the current through the inductor. If the initial charge on the capacitor is q_0, and the initial current is zero, find $q(t)$ as a function of time.