# PHYS2502 Mathematical Physics Homework #2 Due 25 Jan 2022

*This homework assignment is due at the start of class on the date shown. You can hand it in during class time, or email a PDF of your completed assignment to the instructor or grader, so that it arrives before the start of class.*

**(1)** The energy of a simple harmonic oscillator made of a mass $m$ and a spring with stiffness constant $k$ imoving in one dimension $x$ is $E = mv^2/2 + kx^2/2$, where $v = dx/dt$.

(a) Take the derivative of the right side, along with Newton's Second Law and Hooke's Law, to show that the energy does not change with time.

(b) Integrate over the quarter of a period where both $v$ and $x$ are positive, and derive an expression for the period $T$ in terms of $k$ and $m$. The integral is easy to carry out using a change of variables involving a circular function.

**(2)** A dam in the shape of an inverted triangle blocks a river valley, forming a lake of depth $D$ and width $W$. Taking the water pressure $p(y) = \rho g y$ at depth $y$ from the surface of the lake, find the total force acting on the dam. Check that your result is dimensionally correct. Calculate the force on the Hoover Dam ($W = 200$ m) from Lake Mead ($D = 160$ m). Express your result in tons of force.

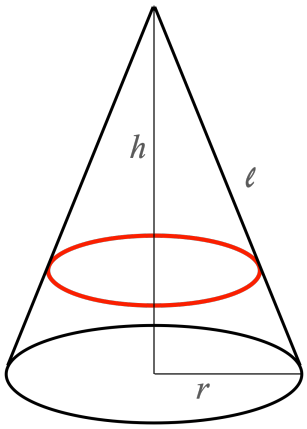
**(3)** Find the derivative of $\tan x \equiv \sin x / \cos x$ with respect to $x$. Then use the change of variables $ax = \tan u$ to evaluate the integral

$$\int_0^\infty \frac{dx}{1 + a^2 x^2}$$

You might want to check your answer using MATHEMATICA.

**(4)** Show that $f(x) = \int_1^x (1/t)dt$ has the property $f(ab) = f(a) + f(b)$ using an appropriate change of integration variables. Hence show that $f(a^n) = nf(a)$ for $n \in \mathbb{Z}^+$.

**(5)** Consider a right circular cone of height $h$ and base radius $r$, as shown on the right. Let $\ell$ be the slant height of the cone.




(a) Find the volume $V$ in terms of $h$ and $r$ by adding up the volume of a bunch of thin circular disks, one of which is shown in red.

(b) Now find the ratio $h/r$ that maximizes the volume of the cone for a fixed slang length $\ell$.