PHYS2502 Mathematical Physics Homework #1 Due 18 Jan 2022

This homework assignment is due at the start of class on the date shown. You can hand it in during class time, or email a PDF of your completed assignment to the instructor or grader, so that it arrives before the start of class.

(1) Show that the product of pressure P and volume V has the dimension of energy. Use this and the ideal gas law $PV = NkT$, where N is the number of gas molecules and T is temperature in Kelvin, to find the SI units of Boltzmann's constant k.

(2) A simple harmonic oscillator is constructed from a mass m connected to a spring with stiffness k. The stiffness is determined by measuring the force from the spring when it is extended or compressed a certain distance, with the force being proportional to that distance.

(a) For classical oscillations with (position) amplitude A, use dimensional analysis to find the energy scale in terms of m, k, and A.

(b) For quantum mechanical oscillations, the amplitude is not well defined, but we expect the energy scale to depend also on $\hbar$, which has units of angular momentum. Find the energy scale in terms of m, k, and $\hbar$.

(3) The "Planck Length" ℓ_P is the distance at which gravity is unified with quantum mechanics and relativity. Find an expression for ℓ_P in terms of G, $\hbar$, and c. Evaluate it numerically and compare it to the size of the proton.

(4) In class we showed that the derivative with respect to x for $f(x) = x^n$, where n is a positive integer, is $f'(x) = nx^{n-1}$.

(a) Show that this relation also holds for $n = 0$.

(b) Use the definition of the derivative to show that this relation also holds when n is a negative integer.

(c) Use (a) and (b) to show that this relation still holds if $f(x) = x^{p/q}$ where p and q are integers, that is, when the exponent is a rational number. *Hint: Consider* $y^q = x^p$.

(d) Can you use all this to rationalize that the derivative of x^α is $\alpha x^{\alpha-1}$ for any $\alpha \in \mathbb{R}$?

(5) A particle moves in a circle in the (x, y) plane, centered on the origin. Find an expression that relates the velocity $v_x = dx/dt$ in the x-direction and the velocity $v_y = dy/dt$ in the y-direction to the position coordinates x and y. Draw a picture of a circle and indicate a few points on it that convince you that your answer is correct. Of course, you need to explain your reasoning.