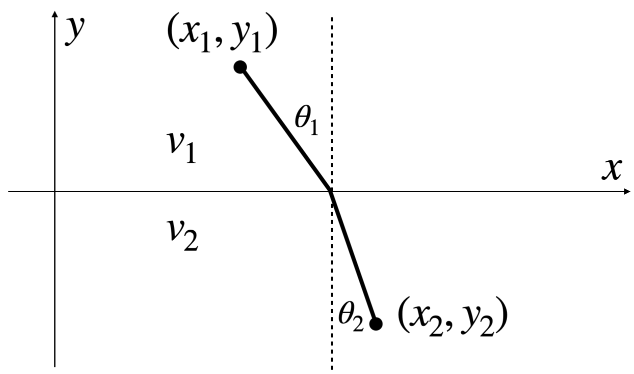
Name: ____________________

PHYS2502 Mathematical Physics S23 Quiz #13 20 Apr 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

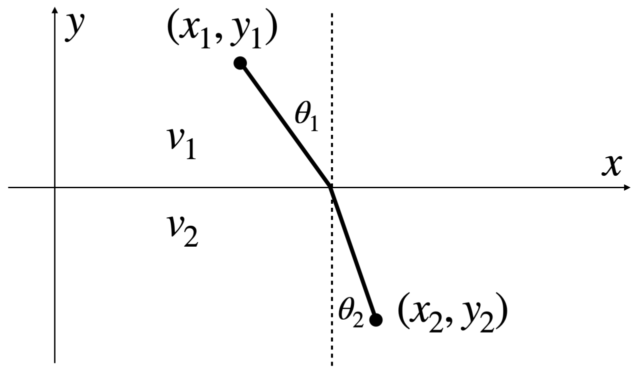
A lifeguard stands on a beach at point (x_1, y_1) and spots a swimmer in trouble in the water at point (x_2, y_2). She runs on the sand with speed v_1, and swims with a speed v_2. To save the swimmer, she wants to minimize the time it takes to get to him.



If the coastline coincides with the x-axis above, find the path of least time. Express your answer in terms of the angles θ_1 and θ_2 and the speeds v_1 and v_2. *Hints: The result should look familiar to you. Do not try to solve this problem using the Euler-Lagrange equation.*

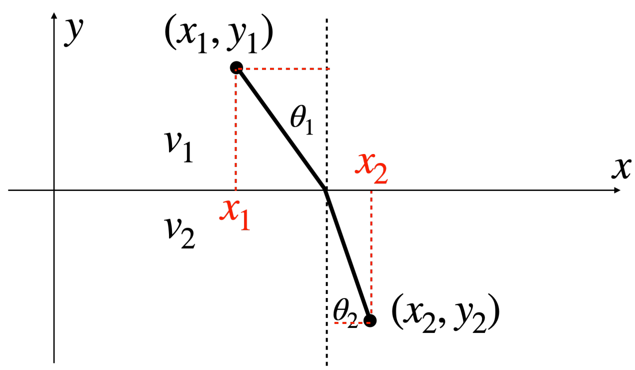
PHY2502 Mathematical Physics Quiz #13 Solution

A lifeguard stands on a beach at point (x_1, y_1) and spots a swimmer in trouble in the water at point (x_2, y_2). She runs on the sand with speed v_1, and swims with a speed v_2. To save the swimmer, she wants to minimize the time it takes to get to him.



If the coastline coincides with the x-axis above, find the path of least time. Express your answer in terms of the angles θ_1 and θ_2 and the speeds v_1 and v_2. *Hints: The result should look familiar to you.* *Do not try to solve this problem using the Euler-Lagrange equation.*

Refer to this annotated version of the figure:



The time it takes to get from point 1 to point 2 is

$$T(x) = \frac{1}{v_1}\left[(x - x_1)^2 + y_1^2\right]^{1/2} + \frac{1}{v_2}\left[(x_2 - x)^2 + y_2^2\right]^{1/2}$$

In order to minimize $T(x)$ we need

$$\frac{dT}{dx} = \frac{1}{v_1}\frac{2(x - x_1)}{\left[(x - x_1)^2 + y_1^2\right]^{1/2}} - \frac{1}{v_2}\frac{2(x_2 - x)}{\left[(x_2 - x)^2 + y_2^2\right]^{1/2}} = 0$$

We recognize the ratios as the sines of the two angles. That is

$$\frac{1}{v_1}\sin\theta_1 - \frac{1}{v_2}\sin\theta_2 \qquad \text{or} \qquad \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

In geometric optics, this is known as Snell's Law of Refraction. In that case, the speed of light is given by c/n in a medium with index of refraction n.